

Roll No.

Total No. of Questions : 08]

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M. Tech.

ADVANCED MATHEMATICS

SUBJECT CODE : PEE - 507 / ELE - 507

Paper ID : [E0487]

[Note : Please fill subject code and paper ID on OMR]

Time : 03 Hours

Maximum Marks : 100

Instruction to Candidates:

- 1) Attempt any Five questions.
- 2) All questions carry equal marks.

Q1) a) Using Laplace transform, evaluate $\int_0^t e^{-t} \cdot \cos t \cdot dt$

b) Using convolution theorem, find $L^{-1} \left\{ \frac{1}{s^2 (s^2 + a^2)} \right\}$.

Q2) (a) Find the Z-transform of ${}^{n+p}C_p$.

(b) If $U(z) = \frac{2z^2 + 3z + 12}{(z-1)^4}$; find the value of u_2 and u_3 .

Q3) (a) Write a note on 2 Dimensional Fast Fourier Analysis.

(b) Find the discrete Fourier transform of $u = \{\sin(ja)\}_{j=0}^{N-1}$.

Q4) (a) Solve the system of equations

$$\frac{dx}{dt} + y = \sin t, \quad \frac{dy}{dt} + x = \cos t$$

given that $x = 2$ & $y = 0$ when $t = 0$

(b) Write a note on matrix representation and state variable approach of difference equation.

Q5) (a) Determine whether the system $x' = -y^2$; $y' = 3x + 2x^3$ has a closed trajectory?

(b) Consider the system $X' = AX$, where $A = \begin{bmatrix} 1 & -3 \\ 2 & -1+\epsilon \end{bmatrix}$ show that when $\epsilon = 0$, the critical point is a centre, stable but not asymptotically stable.

Q6) (a) Transform the BVP, into an integral equation

$$y'' - \sin x y' + e^y = x \text{ given that } y = 1, \frac{dy}{dx} = -1 \text{ when } x = 0.$$

(b) Solve the integral equation: $\int_0^x y(t) y(x-t) dt = 4 \sin 9x$

Q7) (a) If $f(x) = \begin{cases} 12x^3 - 21x^2 + 10x; & 0 \leq x \leq 1 \\ 0 & \text{; otherwise} \end{cases}$

find $P\left(X \leq \frac{1}{2}\right)$ and $P\left(X > \frac{1}{2}\right)$.

(b) A variate X has the probability distribution:

x	-3	6	9
$P(X = x)$	1/6	1/2	1/3

Find $E(X)$, $E(X^2)$ and $E(2X + 1)^2$.

Q8) (a) Two dice are thrown until a 7 is obtained. Find the most probable number of throws and also the expected number of throws.

(b) If X and Y are independent binomial variates $b\left(5, \frac{1}{2}\right)$ and $b\left(7, \frac{1}{8}\right)$ respectively.

Find $P(X+Y=3)$.

