Paper ID [CS203]

(Please fill this Paper ID in OMR Sheet)

B.Tech. (Sem. - 3rd/4th)

MATHEMATICS - III (CS - 203)

Time : 03 Hours **Maximum Marks : 60**

Instruction to Candidates:

1) Section - A is **Compulsory**.
2) Attempt any **Four** questions from Section - B.
3) Attempt any **Two** questions from Section - C.

Section - A

Q1) *(10 × 2 = 20)*

a) State Taylor's Expansion.

b) State and prove second shifting property of Laplace transforms.

c) Find the inverse Laplace transform of $2s/4s^2 + 16$.

d) State Cauchy's integral theorem.

e) Show that function $|z|^2$ is not analytic at any point.

f) Write down one dimensional, two dimensional heat flow equations.

g) Show that if $|z+1| < 1$, $z^{-2} = 1 + \sum_{n=1}^{\infty} (n+1)(z+1)^n$.

h) Find the length of the curve $y = \frac{4}{3} x^{3/2}$ for $0 \le x \le 20$.

i) State and prove sufficient condition for a function to be analytic.

j) Determine a,b,c,d so that function $f(z) = (x^2+axy+by^2) + i(cx^2+dxy+y^2)$ is analytic.

Section - B

(4 × 5 = 20)

Q2) Expand $f(z) = \dfrac{1}{z^2(z-i)}$ as a Laurent's series about i and hence find the residue. There at.

Q3) Evaluate $\oint_C \dfrac{z-23}{z^2-4z-5}dz$, where C is the circle $|z-2| = 4$

Q4) Find the image of circle $|z-1| = 1$ in the w-plane under the mapping $w = z^2$.

Q5) Determine the analytic function whose real part is $e^x(\cos y - y \sin y)$.

Q6) Verify the Roll's theorem to the function $f(x) = e^{-x} \sin x$, $x \in [0, \pi]$.

Section - C

(2 × 10 = 20)

Q7) Evaluate $\int_{-\infty}^{\infty} \dfrac{\cos x}{x^2+a^2}dx$.

Q8) solve $\nabla^2 u = 0$, under the condition $(h = k = 1)$,

$u(0,y) = 0$, $u(4,y) = 12 + y$, for $0 \le y \le 4$;

$u(x,0) = 3x$, $u(x,4) = x^2$ for $0 \le x \le 4$.

Q9) A string of length 'ℓ' is initially at rest in equilibrium position and each of its points is given the velocity $\left(\dfrac{\partial y}{\partial t}\right)_{t=0} = b \sin^3 \dfrac{\pi x}{\ell}$. Find the displacement $y(x,t)$.