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ACTIVE SEISMIC EARTH PRESSURE DUE TO UNSATURATED BACKFILL

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ABSTRACT

In earthquake prone areas, estimation of accurate seismic earth pressure due to backfill is an important factor governing the overall stability of earth retaining structures. In practice, pseudo-static force based limit equilibrium approaches, for example Mononobe-Okabe analysis, are commonly used to determine seismic earth pressure. It is noteworthy that such pseudo-static methods are primarily an extension of Coulomb's wedge theory for computing static earth pressure and gave a solution considering the dry cohesionless backfill only. In pseudo-static methods, seismic effects are usually accounted by introducing two additional time-independent invariable inertial forces computed using horizontal and vertical seismic acceleration coefficients. Further, a considerable literature is available suggesting the modification of pseudo-static method to compute seismic earth pressure due to c-ϕ backfill. Essentially, such analytical approaches are based on determining pressure due to soil weight, inertial forces, surcharge and cohesion which are then optimized with respect to the potential failure plane.

It is well understood that cohesion has a tendency to reduce earth pressure and the dynamic component does not depend on cohesion. Also, it is evident that a limited literature is readily available on effect of saturation of backfill on seismic earth pressure. Consequently, a study focused on unsaturated cohesive soil as the backfill material will be a significant contribution to both geotechnical professionals dealing with earth retaining structures and sparsely available literature. In this paper, an analytical solution is developed for active seismic earth pressure on the back of a retaining wall supporting an unsaturated c-ϕ backfill and considering both horizontal and vertical seismic coefficients. On formulation of an equation using Pseudo Static technique, two parameters namely, Inertial parameter $(K_\gamma)_{dyn}$ and Cohesive parameter $(K_c)_{dyn}$, were obtained. A minima is obtained is case of the $(K_c)_{dyn}$; and a maxima is obtained in case of $(K_\gamma)_{dyn}$ values, when plotted for different wedge angles. $(K_c)_{dyn}$ is not seen to vary for dynamic and static cases as it is not influenced by k_v and k_h; whereas $(K_\gamma)_{dyn}$ varies for dynamic and static case, because of inertial dependence on k_v and k_h. As ϕ increases $(K_\gamma)_{dyn}$ is seen to decrease for both k_v and k_h values. $(K_\gamma)_{dyn}$ increases for the values as both horizontal and vertical inertial component increase, showing a direct dependence; whereas $(K_c)_{dyn}$ remains constant. Due to unsaturated soil, matric

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suction changes the behavior of backfill. The effect of matric suction on cohesion of soil and on tension zone is incorporated. A retaining wall with vertical back and supporting a horizontal backfill with a planar failure surface has been considered for the analysis. The effects of matric suction, soil friction angle, wall friction angle, horizontal and vertical earthquake acceleration components on the active earth pressure have been explored Parametric study was done to understand the effects of different soil and wall parameters on the variation of seismic active earth pressure coefficients. Cohesion is a parameter that caused a big change in $(K_c)_{dyn}$ values.The effect of unsaturation on pressure, $(K_\gamma)_{dyn}$ and $(K_c)_{dyn}$ was studied. As the change in specific weight of soil due to unsaturation was neglected so the $(K_\gamma)_{dyn}$ didn't change; whereas $(K_c)_{dyn}$ values decreased due to increase in effective cohesion. The increase in effective cohesion, in turn, decreased the pressure values exponentially. This shows that neglecting matric suction in calculation can cause a large variation in values of active seismic earth pressure behind a cohesive unsaturated backfill.

Keywords: active earth pressure, pseudo-static analysis, unsaturated backfill, c-ϕ soil, matric suction

ACTIVE SEISMIC EARTH PRESSURE DUE TO UNSATURATED BACKFILL

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ABSTRACT: In this paper, an analytical solution is developed for active seismic earth pressure on the back of a retaining wall supporting an unsaturated c-ϕ backfill. A retaining wall with vertical back and supporting a horizontal backfill with a planar failure surface has been considered for the analysis. The effects of matric suction, soil friction angle, wall friction angle, horizontal and vertical earthquake acceleration components on the active earth pressure have been explored. The observations of the analysis quantified that the matric suction being additive in nature to the cohesion decreases the earth pressure even further.

INTRODUCTION

The force exerted on a retaining structure, by soil retained at an angle. is known as Earth Pressure. The magnitude and classification of earth pressure depends upon the movement of the soil and the structure. Depending upon the wall movement, there are three possibilities, namely,

- Earth pressure at Rest (P_o)
- Movement away from fill – Active Earth Pressure (P_a)
- Movement towards the fill – Passive Earth Pressure (P_p)

In static condition, these states are obtained by mainly Rankine's and Coulomb's thoeries. However, under dynamic conditions, the retaining structures are subjected to dynamic motion and consequently owing to ground motion the dynamic earth pressure is more than the static earth pressure. Mononobe and Matsuo [1,2] and Okabe [3] have been widely used for estimation of seismic earth pressure on a rigid retaining wall. The Mononobe-Okabe (M-O) method is a Psuedo-static approach which incorporates seismic accelerations in the form of inertial forces.

M-O METHOD

The original M-O method did not account for cohesion and friction angle, but several authors have extended the M-O method to incorporate c–ϕ soil. According to M-O analysis the total seismic force acting on wall can be expressed as

$$P_{ae} = 1/2\gamma H^2 (1-k_v) K_{ae} \qquad (1)$$

where K_{ae} is the seismic coefficient of lateral earth pressure, given by

$$K_{ae} = \frac{\sin(\alpha-\phi+\theta)\cos(\alpha-\beta)[\cos(\beta-i)+\frac{2c}{\gamma H(1-k_v)}\cos(\beta)]}{\cos^2\beta\cos\theta\sin(\alpha-i)\cos(\alpha-\beta-\phi-\theta)} - \frac{2c}{\gamma H(1-k_v)}\frac{\cos(\beta-i)\cos(\phi)}{\cos(\beta)\sin(\alpha-i)\cos(\alpha-\beta-\phi-\theta)} \qquad (1a)$$

Eq. 1 becomes indefinite for $k_h > \tan(\phi) + \frac{2c}{\gamma H}$ A similar restriction is common to other solutions as seen in Prakash & Saran[4]; Richards & Shi [5]. For the case of cohesive soils, the general form of the seismic coefficient can be written as $K_{ae} = N_{a\gamma} + \frac{2c}{\gamma H} N_{ac}$, where $N_{a\gamma}$ and N_{ac} are dimensionless earth pressure factors. These parameters need to be optimized to determine the maximum load. Prakash and Saran [4], Saran and Prakash [6] and Saran and Gupta [7] proposed a solution to account for surface cracks and wall adhesion. They proposed a solution for seismic earth pressure on a retaining wall supporting c–ϕ soils, in which the contributions of soil weight and cohesion are optimized separately, in some cases leading to two distinct failure planes.

The above two methods imply the existence of multiple failure surfaces, since the coefficients $N_{a\gamma}$ and N_{ac} are optimized separately. In all Coulomb type of solutions, only force equilibrium is used; and therefore, the distribution of the lateral thrust is not determined. An important disadvantage of these methods is the lack of experimental data at high accelerations. Shukla et al.[8] developed an expression for the total seismic active force on a retaining wall supporting c–φ backfill based on the Coulomb sliding wedge concept, for the case of smooth vertical walls with flat backfills and no surcharge and flat backfills.

This study works on the general solution of total earth pressures for a cohesive soil behind a vertical backfill and compares the results with the results obtained from Prakash and Saran, [4] and Saran and Prakash, [6].

PROPOSED ANALYTICAL FORMULATION

A vertical retaining wall, consisting of a horizontal c–φ backfill of specific weight γ behind the vertical face AB of height H is shown in Fig. 1. A trial failure wedge ABC is assumed and the failure occur along the plane BC which propagates at an angle λ to the horizontal. Tension crack width extends to a depth z below the top surface of the backfill. The horizontal and vertical seismic inertial forces, k_hW and k_vW are also applied to the trial wedge. The outward direction of the horizontal inertia component is assumed to be the positive dynamic active thrust and to be the most critical case. Both vertically downward and upward cases have been considered for vertical inertial force component considering positive and negative signs, respectively. Here k_h and k_v are the horizontal and vertical seismic coefficients, respectively. C is the total cohesive force on the failure plane BD’, and Ca is the total adhesive force mobilised along the wall-backfill interface AB.

From geometry of Fig. 1,

$$DB = H\sec(\alpha) \tag{2}$$

$$AD = H\tan(\alpha) \tag{3}$$

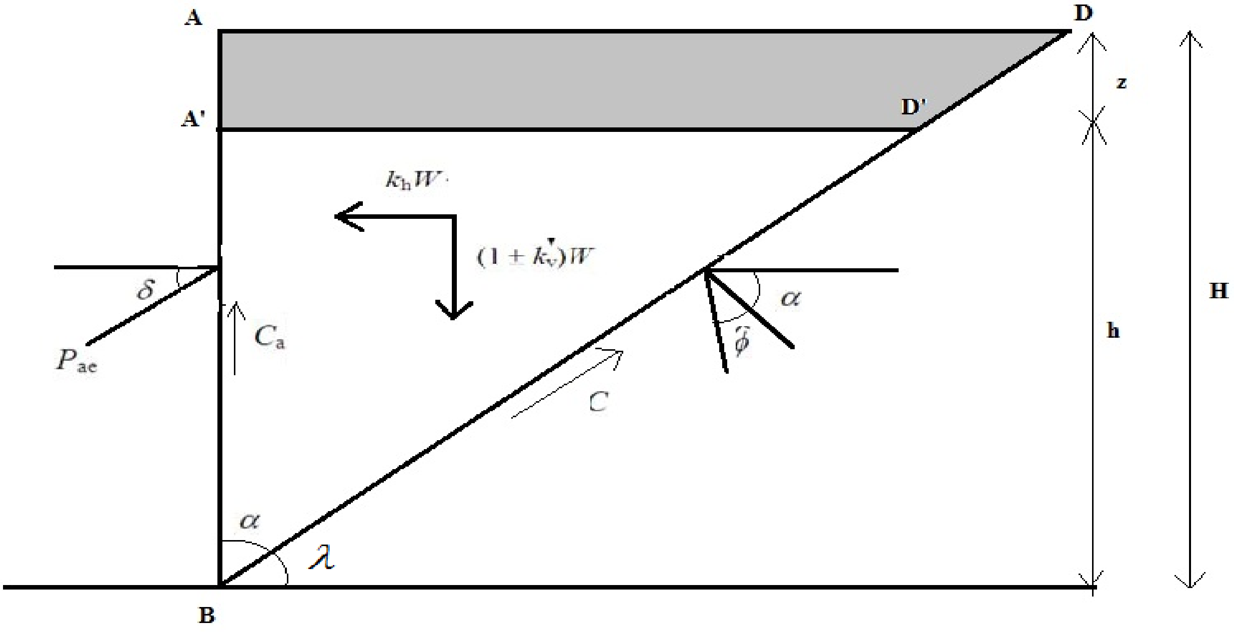

Fig 1. Various forces acting on the trial wedge

Now, considering the tension crack width ‘z’ the new dimensions can be written as,

$$D'B = H\sec(\alpha)(1-\frac{z}{H}) \qquad (4)$$

It is generally assumed that the mobilized cohesive resistance within the tension crack zone varies linearly from c at the bottom of the tension crack to zero at the top of the tension crack [9]. Here, the cohesion is assumed to be constant throughout, so the average value of cohesion and adhesion is not taken. The tension crack width ‘z’ is given by

$$z = \frac{2C}{\gamma\sqrt{K_\phi}} \qquad (5)$$

Where, $K_\phi = \dfrac{1-\sin(\phi)}{1+\sin(\phi)}$

The different forces on the trial wedge can be summarised as in the table 1 below.

Table 1. Summary of forces on Trial Wedge

Force	Vertical component	Horizontal component
Weight	$+W(1\pm k_v)$	$+W(k_h)$
Cohesion	$-C(D'B)\sin(\lambda)$	$-C(D'B)\cos(\lambda)$
Adhesion	$-C_a(A'B)$	-
Pressure	$-P\sin(\delta)$	$-P\cos(\delta)$
Reaction	$-R\sin(\phi+\alpha)$	$+R\cos(\phi+\alpha)$

The vertical components which are upwards are taken as negative and the components which are downwards are taken as positive. The horizontal components which are outwards, away from backfill, are taken as positive and the components which are inwards to backfill are taken as positive. Using $\sum F_v = 0$ and $\sum F_h = 0$, we get

$$W(1\pm k_v) = C(D'B)\sin(\lambda) + C_a(A'B) + P\sin(\delta) + R\sin(\phi+\alpha) \qquad (6)$$

$$W(k_h) = C(D'B)\cos(\lambda) + P\sin(\delta) + R\cos(\phi+\alpha) \qquad (7)$$

Multiplying (6) by $R\cos(\phi+\alpha)$ and multiplying (7) by $R\sin(\phi+\alpha)$, we can eliminate the reaction component and we will be left with only weight, cohesion, adhesion and pressure components. The final equation in terms of P is given as

$$P = \frac{[W(1\pm k_v)\cos(\phi+\alpha) + W(k_h)\sin(\phi+\alpha)]}{\sin(\phi+\alpha+\delta)} - \frac{[C(D'B)\sin(\lambda+\alpha+\phi) + C_a(A'B)\cos(\phi+\alpha)]}{\sin(\phi+\alpha+\delta)} \qquad (8)$$

Putting the values of W, A’B and D’B, we get an Eq (8) which is similar to the equation obtained by Saran and Prakash.

$$P = \frac{1}{2}\gamma H^2\left[\frac{(1\pm k_v)\tan(\alpha)\cos(\phi+\alpha) + k_h\tan(\alpha)\sin(\phi+\alpha)}{\sin(\phi+\alpha+\delta)}\right] - CH\left[\frac{\cos(\phi+\alpha) + (1-\frac{z}{H})\sec(\alpha)\sin(\lambda+\alpha+\phi)}{\sin(\phi+\alpha+\delta)}\right] \qquad (9)$$

Introduction the following dimensionless parameters:

$$(K_\gamma)_{dyn} = \frac{(1\pm k_v)\tan(\alpha)\cos(\phi+\alpha) + k_h\tan(\alpha)\sin(\phi+\alpha)}{\sin(\phi+\alpha+\delta)} \qquad (10)$$

$$(K_c)_{dyn} = \frac{\cos(\phi+\alpha) + (1-\frac{z}{H})\sec(\alpha)\sin(\lambda+\alpha+\phi)}{\sin(\phi+\alpha+\delta)} \qquad (11)$$

Eq (8) can be written as

$$P = \frac{1}{2}\gamma H^2(K_\gamma)_{dyn} - CH(K_c)_{dyn} \qquad (12)$$

Where, $(K_\gamma)_{dyn}$ and $(K_c)_{dyn}$ are dimensional parameters dependent upon various soil and wall characteristics.

PARAMETRIC STUDY

Comparison of $(K_\gamma)_{dyn}$ and $(K_c)_{dyn}$

$(K_\gamma)_{dyn}$ and $(K_c)_{dyn}$ so obtained have been compared with the results from Saran et.[6], Monobobe okabe [2] and static pressure coefficients. These parameters are dependent upon $\alpha, z, \lambda, \delta, \phi$; and when plotted for a specific set of soil and wall parameters these show a peculiar variation with respect to the failure wedge angle α. Saran et. reported similar coefficients as;

$$(N_c)_{dyn} = \frac{\cos(\phi+\alpha)+\cos(\phi)\sec(\alpha)}{\sin(\phi+\alpha+\delta)} \tag{13}$$

$$(N_\gamma)_{dyn} = \frac{[(n+0.5)\tan(\alpha)][(\cos(\alpha+\phi)+k_h\sin(\phi+\alpha)]}{\sin(\phi+\alpha+\delta)} \tag{14}$$

A wall of height 8 m is assumed, containing a soil of parameters as; γ is 18 kN/m^3, ϕ is 22°, cohesion is 5kN/m^2. Comparative variation of inertial parameter is shown in Fig. 2 and cohesive parameter is shown in Fig. 3.

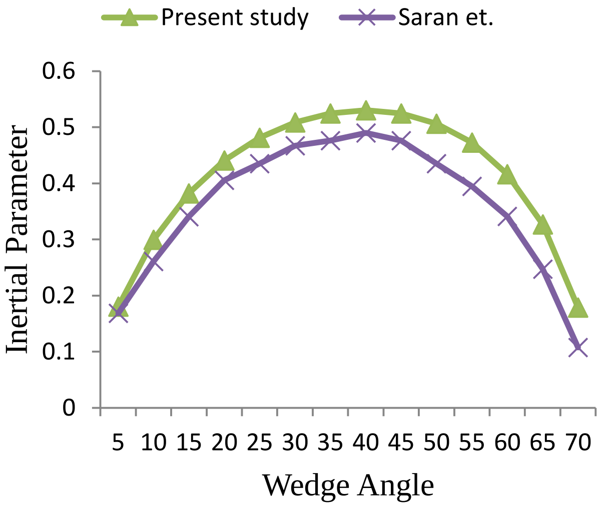

Fig. 2 Variation of inertial parameter, $(K_\gamma)_{dyn}$ with α

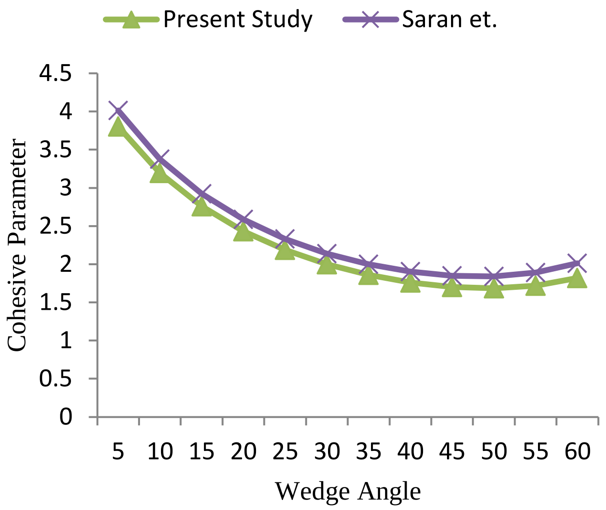

Fig. 3 Variation of Cohesive parameter, $(K_c)_{dyn}$ with α

A minimum is obtained is case of the $(K_c)_{dyn}$; and a maxima is obtained in case of $(K_\gamma)_{dyn}$ values. If for such case k_v and k_h are put to zero, the dynamic earth pressure coefficients thus become the static pressure coefficients. $(K_c)_{dyn}$ is not seen to vary for dynamic and static cases as it is not influenced by k_v and k_h; whereas $(K_\gamma)_{dyn}$ varies for dynamic and static case, because of inertial dependence on k_v and k_h. If we assume the cohesion and adhesion to be zero Eq. 9 becomes similar to the Mononobe Okabe equation.

For dynamics case, the variation of inertial component for different methods is presented in the Table 2 below,

Table 2 Comparison of results for different values of k_v and k_h.

k_h	k_v	Present Study	Saran et.	MO
0.1	0.05	.552	.531	.514
0.2	0.05	.859	.847	.784
0.3	0.05	.958	.934	.841
0.1	0.1	.575	.547	.537
0.1	0.2	.619	.593	.582
0.1	0.3	.664	.644	.627
0.1	-0.1	.486	.468	.448
0.1	-0.2	.441	.422	.403

For the statics case, when k_v and k_h are zero we obtain the results from statics equation as depicted in table below

Table 3 Comparison of results for static case .

k_v	k_h	Present study	Saran et.	MO
0.0	0.0	.452	.463	.455

Variation of $(K_\gamma)_{dyn}$ and $(K_c)_{dyn}$ with different soil and wall parameters.

As stated earlier $(K_\gamma)_{dyn}$ and $(K_c)_{dyn}$ depend on various soil and wall parameters. A wall of height 8m is assumed, containing a soil of parameters as; γ is 18kN/m^3, ϕ is 22^o, cohesion is 5kN/m^2. $(K_\gamma)_{dyn}$ denotes the inertial parameter and $(K_c)_{dyn}$ denotes the cohesive parameter. Fig 4 denotes the variation of $(K_\gamma)_{dyn}$ with respect to ϕ for different k_h values. As ϕ increases $(K_\gamma)_{dyn}$ is seen to decrease. $(K_\gamma)_{dyn}$ increases as the value of horizontal inertial component increases showing a direct dependence whereas $(K_c)_{dyn}$ remains constant.

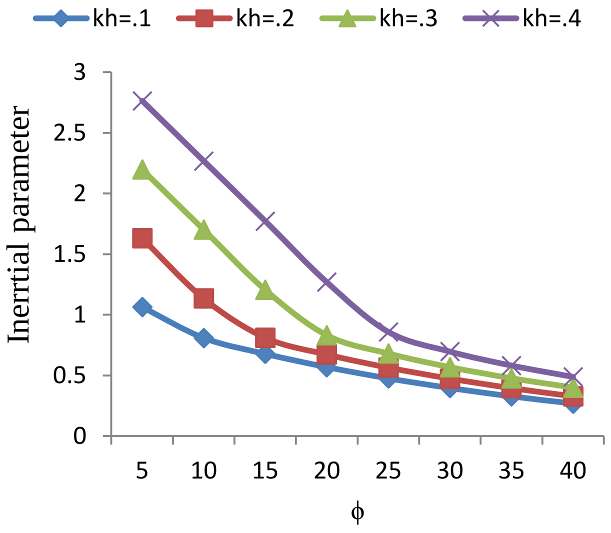

Fig. 4 Variation of inertial parameter, $(K_\gamma)_{dyn}$ with ϕ for different k_h values

Fig 5 denotes the variation of $(K_\gamma)_{dyn}$ with respect to ϕ for different k_v values. $(K_\gamma)_{dyn}$ is also seen to increase as the value of vertical inertial component increases whereas $(K_c)_{dyn}$ remains constant. As the k_v value goes to negative, the vertical inertial force acts upwards hence decreasing the net weight of the soil mass, this in turn decreases the pressure.

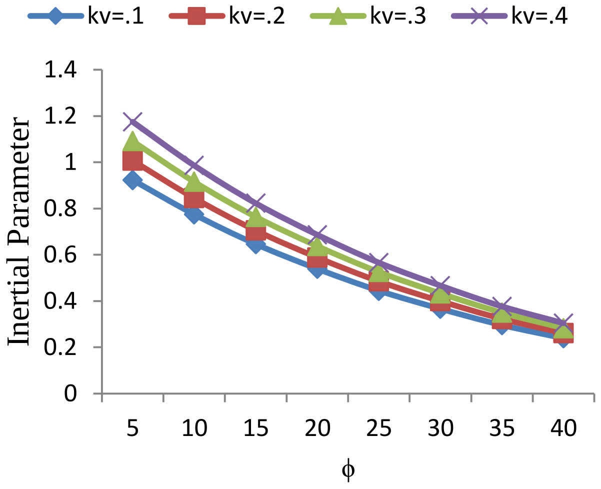

Fig. 5 Variation of inertial parameter, $(K_\gamma)_{dyn}$ with ϕ for different k_v values

Fig 6 and Fig 7 show the variation of $(K_\gamma)_{dyn}$ and $(K_c)_{dyn}$ with friction angle and wall friction angle, δ, respectively. The inertial component is seen to decreases as δ increases. The cohesive component is seen to decrease for lower values of friction angle; but as the friction angle increases the value of cohesive component is seen to increase over the value of cohesive component at lower δ .

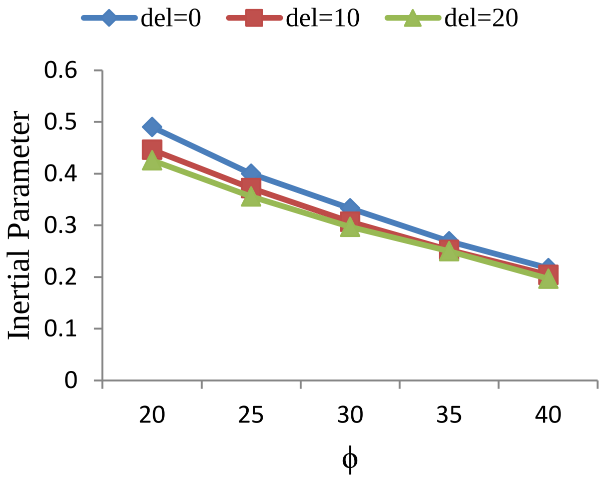

Fig. 6 Variation of $(K_\gamma)_{dyn}$ with ϕ for different δ values

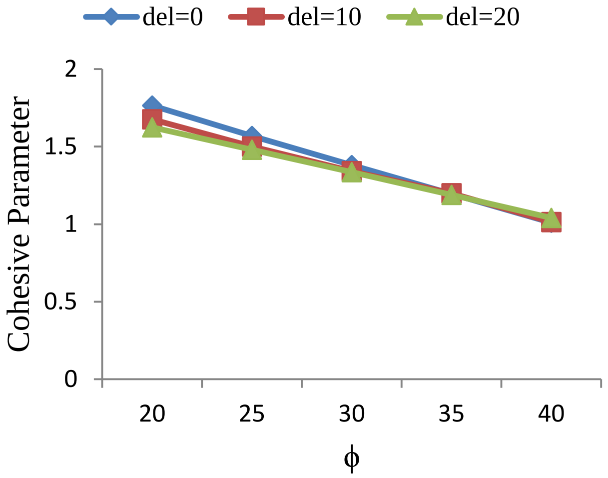

Fig. 7 Variation of $(K_c)_{dyn}$ with ϕ for different δ values

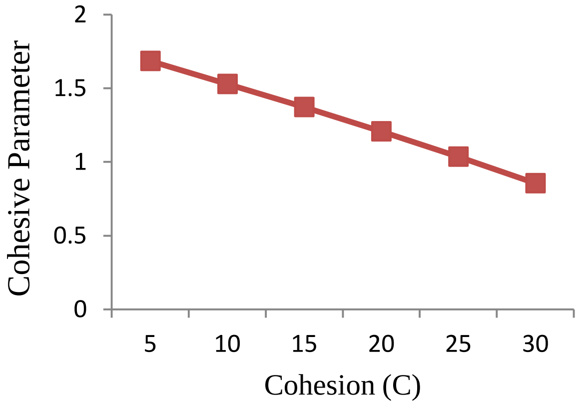

Fig. 8 Variation of $(K_c)_{dyn}$ for different cohesion (C) values

Fig 8 depicts the variation of $(K_c)_{dyn}$ with respect to cohesion C. As the value of cohesion increases the tension crack width increases; this in turn causes the effective value of $(K_c)_{dyn}$ to decrease. Cohesion was seen as an important parameter on which $(K_c)_{dyn}$ is dependent. The net effect of cohesive forces was observed to decrease the effective active earth pressure.

Variation of $(K_\gamma)_{dyn}$, $(K_c)_{dyn}$ and P with saturation of soil backfill.

Any soil with a negative pore-water pressure is considered. to be unsaturated soil It is also recognized that soils with negative pore-water pressures can be saturated or contain air bubbles in an occluded form. The above expressions take care of cohesive fully saturated backfill. A very limited literature is available on how to extend this theory to satisfy the case of unsaturated backfill. Two notable effects of unsaturation in soil are taken and extended in this section;

Change in cohesion of soil

Due to change in saturation of soil, the cohesive properties of soil change. Petersen [10] and Fredlund [11], gave a new cohesion parameter for soils having saturation less than 85%. The cohesion for these soils is written as

$$c^* = c' + c_\psi \qquad (15)$$

Where c' is the effective cohesion and c_ψ is known as effective cohesion due to suction. c_ψ is equal to $(u_a - u_w)\tan\phi^\beta$; where $(u_a - u_w)$ is the matric suction and ϕ^β is the angle indicating the rate of increase of shear strength with respect to matric suction. This generally has a value of 16°-22°.

Change in tension cracks due to matric suction

The change in cohesion and the matric suction causes a change in tension crack width as shown by fredlund. The tension crack width for unsaturated soil is given as;

$$z=\frac{\dfrac{2c'}{\sqrt{K_\phi}}+\dfrac{2f_w(u_a-u_w)\tan\phi^b}{\sqrt{K_\phi}}}{\gamma+\dfrac{2}{H\sqrt{K_\phi}}f_w(u_a-u_w)\tan\phi^b} \qquad (16)$$

Where, $K_\phi=\dfrac{1-\sin(\phi)}{1+\sin(\phi)}$,

f_w is a parameter dependent upon the depth of water table from wall and also on the variation of matric suction. Fig 9 shows the variation of $(K_c)_{dyn}$ with increase in matric suction. As the matric suction increases the $(K_c)_{dyn}$ is seen to dip even lower, this is due to additive effect of matric suction on net cohesion.

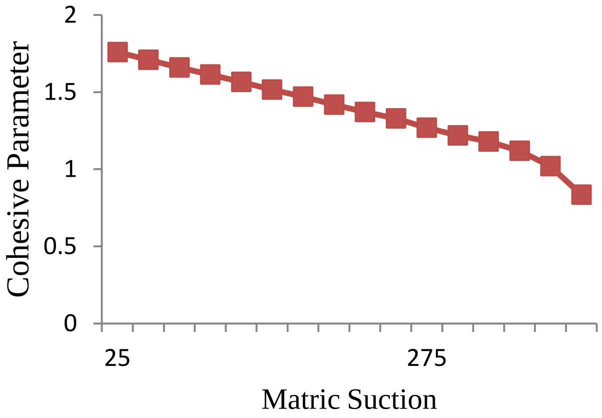

Fig. 9 Variation of cohesive parameter, $(K_c)_{dyn}$ for different matric suction values

Fig. 10 shows the variation of $(K_c)_{dyn}$ with friction angle, and matric suction is put to 50kPa. Matric suction causes a lower value of $(K_c)_{dyn}$ than the value from Eq (12), this is because the change in tension crack due to matric suction.

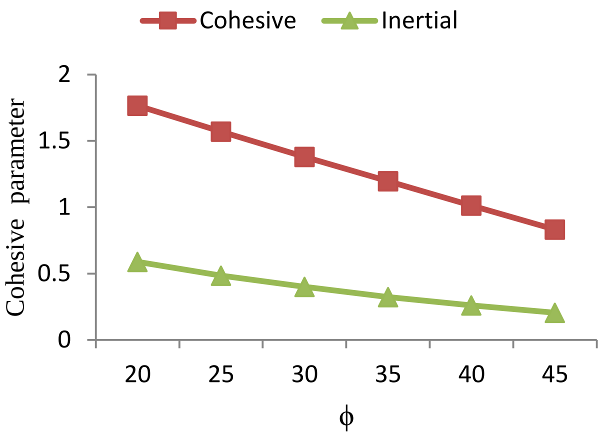

Fig. 10 Variation of Cohesive parameter, $(K_c)_{dyn}$ for different friction value angle, for case of no matric suction and case of matric suction set to 50 kPa

Fig. 11 below show the variation of Pressure with respect to matric suction. A wall of height 8m is assumed, containing a soil of parameters as; γ is 18kN/m^3, ϕ is 22°, cohesion is 5kN/m^2, ϕ^b is 14°. As evident from the Fig. 11 the variation of pressure with change in matric suction provides us with an exponential graph. A variation of just 100kPa in matric suction causes pressure to decrease from 187kPa to 11.9kPa.

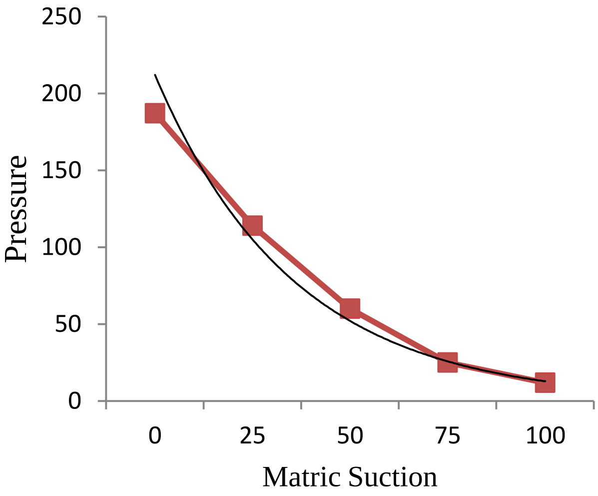

Fig. 11 Variation of active earth pressure with matric suction

CONCLUSIONS

Parametric study was done to understand the effects of different soil and wall parameters on the variation of seismic active earth pressure coefficients. The $(K_\gamma)_{dyn}$ was seen to have a higher value than values obatined from [5] and on the other hand $(K_c)_{dyn}$ was seen to have to lower values. k_v and k_h had a vast change in values of $(K_\gamma)_{dyn}$ whereas $(K_c)_{dyn}$ was constant throughout. Cohesion is a parameter that caused a big change in $(K_c)_{dyn}$ values.The effect of unsaturation on pressure, $(K_\gamma)_{dyn}$ and $(K_c)_{dyn}$ was studied. As the change in specific weight of soil due to unsaturation was neglected so the $(K_\gamma)_{dyn}$ didn't change; whereas $(K_c)_{dyn}$ values decreased due to increase in effective cohesion. The increase in effective cohesion, in turn, decreased the pressure values exponentially. This shows that neglecting matric suction in calculation can cause a large variation in values of active seismic earth pressure behind a cohesive unsaturated backfill.

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