A TOTAL STRESS MODEL FOR UNDRAINED CYCLIC BEHAVIOUR OF REMOULDED CLAY

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ABSTRACT: The cyclic behaviour of remoulded kaolin samples is first examined in this paper. Based on the laboratory test results, a non-linear hysteretic constitutive model was prescribed. The suite of laboratory triaxial tests and resonant column tests performed in this study encompass small to moderately large strains with cyclic frequencies varying from 0.05 to 50 Hz. The results of the laboratory element tests were used to calibrate the proposed hyperbolic-hysteretic constitutive model. Despite the relatively simple total stress framework, numerical analyses performed using this model was able to reasonably replicate the modulus reduction and cyclic stiffness degradation of the kaolin clay.

INTRODUCTION
During earthquake events, clayey soils may be subjected to undrained cyclic loading. While many constitutive models have been developed for sands under cyclic loading [1], there are fewer constitutive models for soft clays subjected to such loadings. Much of the research to date deal with specific aspects of the constitutive behaviour of soft clays under cyclic loading such as low-amplitude shear modulus [2,3], modulus reduction and damping ratio variation with strain [4,5,6,7,8], stiffness and strength degradation under cyclic loading [9,10] as well as effective stress and pore pressure response [11]. In spite of these, certain aspects of soft clay behaviour under cyclic loading remain inconclusive. For instance, few researchers reported a decrease in the damping ratio of Bangkok Clay with increasing loading frequency [12]. On the other hand, others reported that the damping ratio of a low-plasticity compacted clay increases with frequency [13].

This paper discusses the performance of a non-linear hysteretic model for the stress-strain behaviour of normally consolidated kaolin clay under undrained triaxial loading conditions. The dynamic behaviour of kaolin clay was first investigated using strain-controlled undrained cyclic triaxial tests and resonant column tests. Using key features identified in the laboratory test results, a simple non-linear model was proposed based on a hyperbolic stress-strain relationship which incorporates the generalized Masing’s rules [14] and the concept of degradation index [9]. The validation of the proposed model was conducted by back-analyzing undrained cyclic triaxial tests using the general purpose finite element program ABAQUS 6.7.1, with the non-linear soil model incorporated as a user-defined material.

CYCLIC TRIAXIAL AND RESONANT COLUMN TESTS
The suite of laboratory cyclic triaxial tests and resonant column tests performed in this study encompass very small to moderately large strains (0.002% to 1.37%), with cyclic frequencies varying from 0.05 to 50 Hz. The tests were done for two confining stresses such as, 200 and 150 kPa.

Maximum Modulus
Figure 1 shows the variation in shear modulus obtained from the resonant column tests for two mean effective stress levels $p'$, together with the maximum shear moduli estimated using [3]’s relation for normally consolidated speckled kaolin. As can be seen, the maximum shear modulus was about 5% higher than Viggiani and Atkinson’s predicted maximum shear modulus.

![Shear Moduli](image1)

Fig. 1 Shear moduli from resonant column tests for two different mean effective stress levels

The measured data can be fitted by the following power law

\[ G_{\text{max}} = \frac{2060 (p')^{0.63}}{p_{r}} \]

in which $p'$ is the mean effective stress in kPa, $G_{\text{max}}$ the maximum or initial shear modulus, also in kPa and $p_{r}$ the reference pressure, herein taken to be 1 kPa.

Effect of Shear Strain Amplitude
Values of the secant shear modulus $G_{\text{sec}}$ obtained from cyclic triaxial (1st cycle) and resonant column tests conducted in this study, normalized by the maximum shear modulus.
modulus $G_{\text{max}}$, are shown on Figure 2. The reverse-S trend exhibited by the results has been reported in many previous studies [6,7,4,15,2]. For the kaolin clay used in this study, a significant decrease in the secant modulus was observed when the shear strain level exceeded about 0.01%.

Fig. 2 Variation of $G_{\text{sec}}/G_{\text{max}}$ with shear strain from present experiments and reported trends

On the other hand, previous studies have shown that the damping ratio (D) in clay usually increases with strain level, forming an ‘S’-shaped curve. As Figure 3 shows, the damping ratio in the first cycle obtained from the present cyclic triaxial and resonant column tests also increases with strain in a similar manner. Furthermore, the current kaolin clay data generally fall between the reported results of [4] and [2]. The figure 2 and 8 also shows the shear modulus and damping ratio computed from the hyperbolic-Masing relationship, which will be discussed in a later section.

Fig. 3 Variation of damping ratio with shear strain from present experiments and reported trends

Effect of Frequency
The effect of frequency on modulus is less conclusive. The results shown on Figure 2 were compiled from resonant column tests conducted at frequencies ranging from 30Hz to 40Hz whereas the cyclic triaxial tests were conducted at frequencies no higher than 1.5Hz. In spite of the large differences no higher than 1.5Hz. In spite of the large differences, the reverse S-trend, suggesting that the strain amplitude has a much more significant effect on the modulus than the frequency. Teachavorasinskun et al. (2002) [12] also reported that, at large strain levels, the effect of frequency on modulus degradation is quite negligible compared to that of strain. As mentioned earlier, previous studies on the influence of frequency on the damping ratio have reported differing

Fig. 4 Variation of damping ratio with frequency from from present experiments and reported trends

The results from the current cyclic triaxial tests, as plotted in Figure 9, do not indicate any significant trend of decrease or increase in the damping ratio with frequency. In spite of their higher frequency, the resonant column tests yield much lower damping ratios, thereby indicating that strain level is a more dominant factor than frequency. As Figure 4 shows, a similar lack of trend is indicated by Kim et al.’s (1991) and Teachavorasinskun et al.’s (2002) data for low to medium plasticity soils [12,13]. The absence of any significant frequency influence on the damping ratio suggests that much of the damping in clay is hysteretic rather than viscous.

HYPERBOLIC-MASING MODEL
The proposed model is based on a hyperbolic backbone curve relating deviator stress and generalized shear strain. Hysteretic behaviour for unloading and reloading is modelled using Masing’s rules while cyclic degradation is modelled using Idriss’s degradation index [9]

Hyperbolic backbone curve
The backbone curve is represented using a hyperbolic relationship of the form

$$ q = q_f \left( 1 - \frac{3G_{\text{sec}}}{3G_{\text{max}}} \right) \left( 1 + \frac{3G_{\text{max}}}{G_{\text{sec}}} \frac{|q_f|}{q_f} \right) $$

where $q$ is deviator stress, $q_f$ is deviator stress at failure, $G_{\text{max}}$ is maximum shear modulus and $\epsilon_f$ is generalized shear strain. The secant shear modulus ($G_{\text{sec}}$) is given by

$$ G_{\text{sec}} = \frac{q_f}{3\epsilon_f} = \frac{G_{\text{max}}}{1 + 3R\epsilon_f} $$

where $R = G_{\text{max}} / q_f$ is termed the normalized modulus, and $\epsilon_f$ and $q_f$ are the maximum strain and the corresponding deviator stress at the reversal point. The function represented by Eq. 3 is plotted on Figure 2, together with the experimental
data. As shown on Figure 2, Eq. 3 plots well within the range of the experimental data.

Masing’s Rules

The generalized Masing’s rules [14] were adopted to model the hysteretic behaviour of the soil during the unloading and reloading phases of each load cycle. Following [16], it can be shown that the damping ratio is given by

\[
D = \frac{2}{\pi} \left[ \frac{2 + 3R_e}{3R_e} \right] \left[ \frac{2(1 + 3R_e)}{3R_e} \right] \left[ \frac{m(1 + 3R_e)}{3R_e} \right]
\]  
(4)

The theoretical damping ratio given by Eq. 4 is plotted on Figure 3 for Kaolin clay with \( p' = 200 \text{ kPa} \) and \( G_{\text{max}} \) obtained from Eq. 1, together with experimental data and previously published damping curves from other researchers. The results indicate that, for shear strain 0.1%, the hyperbolic curve with Masing’s rules appear to underestimate the damping ratio slightly. For strain level between 0.1% and −1%, the damping ratio is reasonably predicted. For larger strains, the damping ratio implied by Masing’s rules starts to exceed measured values significantly. Similar discrepancies have also been noted by [9] and [16].

Stiffness Degradation

For constant-amplitude cyclic loading, Idriss [9] expressed the degradation index \( \delta \) as a function of the number of load cycles \( N \) using the following power law

\[
\delta = N^{-t}
\]  
(5)

in which \( t \) is a degradation parameter which may be obtained from the slope of the log \( \delta \) vs log \( N \) plot. For cases in which the strain is uniform from the \((m+1)^{\text{th}}\) to \( n^{\text{th}} \) cycle, Idriss et al. (1978) proposed an incremental form given by,

\[
\delta_n = \delta_m [1 + \left( m - n \right)^{1/2}]^{-t}
\]  
(6)

in which \( \delta_m \) and \( \delta_n \) are the degradation indices after \( m \) and \( n \) cycles, respectively. This form is useful for non-uniform cyclic loading.

The data from the present cyclic triaxial tests were processed using Eqs. 5 and 6 to obtain the degradation parameter \( t \). Figure 5 plots the degradation parameter \( t \) against the shear strain amplitude, which shows a trend similar to that reported by Idriss et al. [17]. The points can be reasonably fitted by a logarithmic function of the form,

\[
t = 0.0536 \log(\varepsilon_s) + 0.1285
\]  
(7)

where \( \varepsilon_s \) is the generalised shear strain.

In Eq. 7, negative values of \( t \) are obtained for shear strain levels smaller than 0.1%. This is consistent with the threshold strain level reported by [11], below which there is no significant stiffness degradation.

MODEL PERFORMANCE

Figure 19 shows a quadrant of a triaxial specimen discretized using 20-noded brick elements. The proposed model discussed in the previous section, which incorporates the hyperbolic behaviour of Eq. 6 with the key features of Masing’s rules, was implemented in ABAQUS 6.7.1 as a user-defined material subroutine. The main input to the model is \( G_{\text{max}} \), which is obtained using Eq. 1. The degradation parameter \( t \) was assigned based on Eq. 23. The dynamic analysis was carried out by subjecting the top nodes to 60 cycles of a sinusoidally varying displacement. The initial effective confining stress was prescribed as 200 kPa.

Figure 7 plots the measured and computed stress-strain loops for the 1st and 60th cycles for two different strain amplitudes (0.254% and 1.37%) at a loading frequency of 1 Hz. The generally good agreement between the measured and computed responses indicates that the proposed model can reasonably capture the hysteretic behaviour and stiffness degradation of the kaolin clay used in the experiments.

CONCLUDING REMARKS

The test results presented above highlighted several noteworthy features in the behaviour of remoulded soft clay. Damping appears to be largely hysteretic. This implies that effects due to distortion in scaling viscous effects may not be significant. The experimental observations on the shear modulus and damping ratio lay the basis for a simple non-linear hysteretic model for cyclic loading on remoulded soft clay. Good agreement was obtained between the computed and experimental results. The model has several advantages. Firstly, it requires only two parameters: the maximum shear modulus \( G_{\text{max}} \) and the degradation parameter \( t \). Secondly, the degradation index computed by the model is valid for non-uniform cycles. This makes it relatively easy to use for real earthquakes.
Since the proposed model is based on total stress, it is only applicable to undrained loading. However, this may not be a major setback for most fine-grained cohesive soils which are likely to be under undrained conditions during an earthquake event. The model also significantly over-predicts damping for high strain levels; thus one may therefore need to be cautious about applying it to very large earthquakes. Thirdly, the model also does not consider anisotropic and strain softening behaviour. Finally, since it has only been tested on remoulded soft clay, its validity for highly structured soft clay has not been established.

Fig. 7 Comparison of measured and computed stress-strain loops for two different strain amplitudes at a loading frequency of 1 Hz

REFERENCES