EFFECT OF SUBGRADE SHEAR STIFFNESS ON OBLIQUE PULLOUT BEHAVIOR OF REINFORCED SOIL

Shantanu Patra, Doctoral Student, Dept. of Civil Engng., I.I.T. Delhi. E-mail: patrashantanu@gmail.com
J. T. Shahu, Associate Professor, Dept. of Civil Engng., I.I.T. Delhi. E-mail: shahu@civil.iitd.ac.in

ABSTRACT: The kinematics of failure of reinforced structures such as reinforced soil walls, embankments and slopes is such that failure surface intersects the reinforcement obliquely and thus causes pullout of the reinforcement. Hence for the internal stability analysis of these structures, obliquity of the reinforcement force should be accounted for. An analysis is carried out for evaluation of oblique pullout capacity of an inextensible sheet reinforcement resting on a subgrade soil idealized by a two-parameter linear elastic Pasternak model. Localized deformed shape of the reinforcement, mobilized tension in the reinforcement and its direction are evaluated using the present model. The orientation of the reinforcement at the pullout end has been found to be different than the direction of the pullout force and hence inclination factor is defined. The present study removes the drawback of the oblique pullout analysis based on Winkler model by considering the shear interaction of the springs and gives more accurate prediction of end displacement, mobilized tension and its direction. The present model can be used for the pullout analysis of internal stability of reinforced soil walls and slopes.

INTRODUCTION
Failure of reinforced structures such as reinforced soil walls, embankments and slopes is such that reinforcement undergoes an oblique pullout force at the reinforcement-failure surface intersection. It is found from the experimental investigation [1,2] that the orientation of the reinforcement at the vicinity of the failure surface differs from that of the failure surface. Still now no theory is available in the literature to establish the fact. Earlier works by various researchers [3,4] to find out the oblique pullout behavior of reinforced soil fail to explain the localized behavior of reinforced soil as found from the experimental results available in the literature [1,2]. The reason may be use of Winkler model as subgrade soil which impregnate over simplification of the reinforced-soil behavior and thus suffers from the inherent limitation of the Winkler model, the fact that these models did not consider the shear interaction of the soil layer. The present study removes the drawback of the oblique pullout analysis based on Winkler model by considering the shear interaction of the springs and gives more accurate prediction of pullout force, end displacement, mobilized tension and its direction.

FORMULATION
An inextensible sheet reinforcement of normalized length 1.0 unit is resting on a subgrade soil (Fig. 2a) having subgrade stiffness factor $\mu = k_L / \gamma D$ and shear stiffness factor $G_s = GH / \gamma D L$. The reinforcement is subjected to a normalized oblique pullout force $P^* = P / T_{HP}$ where $T_{HP} =$ axial pullout capacity of the reinforcement (at point B where the sliding mass intersects the reinforcement Figs. 1a-d) at an obliquity $\alpha$. The proposed model for the analysis is shown in Fig. 2(b). The soil-reinforcement interface friction is $\phi$ and overburden soil is represented by overburden stress at the top of reinforcement. The amount of tension $T^*$ in the reinforcement at end B is $T_{max}^*$ with an inclination $\theta$. Vertical
displacement \( W \) at ends \( A \) and \( B \) are \( W_A \) and \( W_B \), respectively. The displacement \( W \) and tension \( T' \) at any point along the reinforcement can be calculated by solving the following two governing coupled equations as [5]

\[
T'_{i+1} = T'_i + \frac{0.5}{n} \left[ \frac{\mu W W_L - G^* W_L d^2 W}{dX^2} \left( \frac{\tan \theta_i}{\tan \phi_i} + 1 \right) + 2 \right]
\]

(1)

\[
W_i = \frac{n^2 (W_{i+1} + W_{i-1}) (2T'_i \tan \phi_i \cos^2 \theta_i + G^*)}{\mu + 2n^2 (2T'_i \tan \phi_i \cos^2 \theta_i + G^*)}
\]

(2)

where \( i \) is the number of elements into which the reinforcement strip is divided (i.e., \( \Delta X = 1/n \)). The boundary conditions become:

at \( X = 0 \), \( \frac{dW}{dX} = 0 \) and \( T' = 0 \); and at \( X = 1 \), \( W = 1 \). (3)

Final governing equations for overall equilibrium as (Patra and Shahu 2011)

\[
\tan \alpha = \frac{1}{\cot \phi_i + \frac{\sum_{i=1}^{n} [\mu W W_i + 2 - n^2 G W_i (W_{i+1} - 2W_i + W_{i-1})] \sin \theta_i}{\sum_{i=1}^{n} [\mu W W_i + 2 - n^2 G W_i (W_{i+1} - 2W_i + W_{i-1})] \cos \theta_i}}
\]

(4)

\[
P' = \frac{1}{2 \cos \theta_i} \sum_{i=1}^{n} [\mu W W_i + 2 - n^2 G W_i (W_{i+1} - 2W_i + W_{i-1})] \cos \theta_i
\]

(5)

where \( \theta_i = \tan^{-1} \left[ \frac{\mu W_i (W_{i+1} - W_i)}{2} \right] \) is the value of \( \theta_i \) for element \( i \); and \( \theta_i = (\theta_i + \theta_{i+1})/2 \).

**Solution and Range of parameters**

Elemental equilibrium equations (Eqs. 1 and 2) are solved in conjunction with the boundary conditions (Eq. 3) and overall equilibrium equations (Eqs. 4 and 5) to obtain \( W_i \) and \( T'_i \) at all nodes. A trial and error procedure is adopted for the solution. Ranges of parameters for the analyses are taken as: \( \mu = 50-1000 \) and \( G^* = 0-50 \), interface frictional angle \( \phi = 20-40^\circ \).

**RESULTS**

**Displacement and tension along the reinforcement**

Variation of displacement, \( W \) and tension, \( T' \) with distance, \( X \) at pullout for different values of \( G^* \), \( \mu \), and \( \alpha \) are shown in Fig. 3 and 4 respectively. The effect of shear stiffness \( G^* \) on the displacement profile is such that for the increase of \( G^* \) the displacement is more distributed along the reinforcement while for reducing values of \( G^* \) the displacement is more localized (Fig 3). But the effect of subgrade stiffness on the displacement profile is quite opposite compare to the effect of shear stiffness. In this case the displacements are localized at the pulling end for higher values of \( \mu \) and the displacements progress towards the far end for lower values of \( \mu \). It is further observed that higher the obliquity of the pull \( \alpha \) is steeper the deformation profile is. The displacement \( W \) at any point in the reinforcement decreases with the increase in \( G^* \), and \( \mu \) but increases with increase in \( \alpha \) (Fig. 3).

Tension \( T' \) is developed in the reinforcement due to mobilization of shear stress at the soil reinforcement interface. For increasing \( G^* \) soil shares more load by distributing the load over larger area. Consequently normal stresses developed in the soil reinforcement interface reduce and hence resulting interface shearing resistance decreases. Hence displacement \( W \) and tension \( T' \) generated in the reinforcement reduces. So, tension \( T' \) at any point in the reinforcement decreases with the increasing \( G^* \) (Fig. 4). As the displacement profile becomes more linear for higher values of \( G^* \) tension in the reinforcement becomes more and more linear. For \( G^* = 50 \) reinforcement is almost horizontal (Fig. 3) and tension (Fig. 4) in the reinforcement at the pulling end is almost equal to the axial pullout capacity of the reinforcement. So, conventional assumption of axial pullout capacity of the reinforcement for the analysis of reinforced soil structure is true only for very high values of shear stiffness, and low value of subgrade stiffness and at low obliquity. Whereas for \( G^* = 0 \) a very high value of pullout capacity and mobilized tension in the reinforcement is observed [3] (\( T' \) as high as 6.6 for \( \mu = 50 \); and \( \alpha = 75^\circ \)). Present model gives \( T' \) equal to 1.23 for \( G^* = 10 \); \( \mu = 50 \); and \( \alpha = 75^\circ \) which seems to be more reasonable [3].

Reinforcement force in the form of normalized tension \( T' \) increases almost linearly up to \( X = 0.4-0.85 \) for \( \mu = 50-1000 \), respectively (shown in Fig. 4). As in the axial pullout case and increases sharply thereafter in nonlinear manner. For stiffer subgrade, normalized displacement are highly localized near the right end thus increasing normal stresses on that portion of the reinforcement and resulting in a sharp increase in tension there. In general higher the inclination \( \alpha \) is higher the displacement at the pulling end is, and higher the maximum tension is. So maximum tension generated in the reinforcement depend mainly on lower shear stiffness \( G^* \), higher subgrade stiffness \( \mu \), and steeper inclination \( \alpha \) of the end force.

**Direction of tension along the reinforcement**

The direction of the tensile forces at any point in the reinforcement is along the tangential direction at that point on the deformed shape of the reinforcement. Hence slope of the reinforcement at any point, \( \theta \) denotes the direction of the reinforcement force at that point. Inclination of the reinforcement force is more at the pulling end and becomes horizontal at the far end (shown in Fig. 5). The direction of
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the tensile force at the pulling end $\theta_L$ is more or less horizontal for high values of $G^*$. With the decreasing of shear stiffness $G^*$ inclination $\theta_L$ of the tensile force in the reinforcement increases and becomes equal to the inclination $\alpha$ of the applied pullout force for $G^* = 0$. For higher values of subgrade stiffness $\mu$ change in the direction of the tensile force along the reinforcement is more due to localized deformation of the reinforcement. The direction of the reinforcement force $\theta$ increases with the increase in obliquity $\alpha$ of the pullout force. For $G^*=0, 10,$ and $50; \theta_L = 75^\circ, 24.5^\circ$ and $5^\circ$, respectively at obliquity of the pullout force $\alpha = 75^\circ$ and $\mu = 1000$ (shown in Fig. 5) shows sharp change in end angle $\theta_L$ due to change in $G^*$.

**Inclination factor**

Inclination factor $I_F$ is defined as the ratio of the inclination of the reinforcement $\theta_L$ at final deformed state to the inclination $\alpha$ of the oblique pullout force; i.e. $I_F = \theta_L / \alpha$ (shown in Fig. 2(d)) and $I_F$ quantifies the deformation behavior of the reinforcement under oblique pull.

Fig. 6 shows that inclination factor $I_F$ reduces with the increase in shear stiffness factor $G^*$ which implies that bending of the reinforcement reduces with increase in $G^*$. On the other hand the effect of $\mu$ on $I_F$ is negligible but significant changes in $I_F$ occurs mainly due to change in $G^*$ (Fig. 6).

$I_F$ approaches to 0 value for very high value of $G^*$ ($G^* > 50$) and approaches to 1.0 for very low values of $G^*$ ($G^* = 0$). Physically it implies that the reinforcement near the failure surface remains horizontal for very high values of shear stiffness of soil (conventional method of analysis). But reinforcement at the end deformed more and more as $G^*$ decreases and the reinforcement slope is equal to the obliquity of pullout force [4]. But in practice these angles are never equal and hence $I_F$ lies in between these two extremes (0 and 1) depending on the shear stiffness of soil and reinforcement [1,2] (In present analysis shear stiffness of reinforcement is neglected).

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Figures:

**Fig. 3.** displacement along reinforcement

**Fig. 4.** tension along reinforcement

**Fig. 5.** direction of tensile force along reinforcement

**Fig. 6.** Inclination factor $I_F$ versus $G^*$—effect of $\mu$ and $\alpha$

**Fig. 7.** $P^*$ versus $I_F$—effect of $\mu$ and $\alpha$
One can easily estimate the range of shear stiffness and vice versa from Fig. 6 for a given $\mu$, $\alpha$, and $I_F$; where $I_F$ can be easily estimated in laboratory by observing deformed shape of the reinforcement in a simple shear stress [1] and $\mu$ may be obtained from simple stress strain relationship. Hence pullout capacity $P^*$, horizontal pullout capacity $P^*_H$, and end displacement $W_L$ are plotted with inclination factor $I_F$ (Figs. 7-9). Fig. 7 shows that pullout capacity is increasing with the increase in inclination factor $I_F$ and the effect is more at high obliquity $\alpha$. For $\mu = 50$ and $\alpha = 60^\circ$, $P^*$ increases from 2 to 2.76 when $I_F$ increases from 0 to 1. Fig. 8 shows that the horizontal component of the pullout force $P^*_H$ is almost linearly varying with the inclination factor. For $\mu = 50$ and $\alpha = 60^\circ$, $P^*_H$ increases from 1 to 1.38 when $I_F$ increases from 0 to 1. Effect is more for higher obliquity $\alpha$ and for higher subgrade stiffness $\mu$. Fig. 9 shows the end displacement is almost linearly varying with the inclination factor $I_F$. The effect is less when the obliquity $\alpha$ is less and subgrade stiffness $\mu$ is more.

Realistic values of $W_L$, $P^*$, and $P^*_H$ are obtained for $I_F = 0.5$, $\mu = 50 - 100$ and $\alpha = 60^\circ$ ($W_L = 0.12-0.03$; $P^* = 2.27-2.41$; $P^*_H = 1.13 - 1.2$ as shown in Figs. 7-9).

**CONCLUSIONS**

1. For increasing $G^*$, soil shares more load by distributing the load over larger area. Consequently normal stresses developed in the soil reinforcement interface reduce and hence resulting interface shearing resistance decreases. In the process, displacement at any point $W$ and tension $T^*$ induced in the reinforcement reduce.

2. Pullout capacity $P^*$ increases with the increase in inclination factor $I_F$. So the higher the bending of the reinforcement, the higher will be its oblique pullout capacity $P^*$. Horizontal pullout capacity $P^*_H$, and end displacement $W_L$ also increase with the $I_F$.

3. Conventional assumption of axial pullout capacity of the reinforcement for the analysis of reinforced soil structure is true only for very high values of shear stiffness and low values of subgrade stiffness, and at low obliquity.

4. Present analysis removes the drawback of earlier model [3,4] and more accurately predict the pullout behavior and explain the experimental investigation [1,2].

5. While conventional method gives most conservative value of pullout capacity, the earlier model [4] overpredicts the pullout capacity. However, the real soil behaves in between these two extremes. The present model bridges this gap and gives a more rational and realistic model for oblique pullout behavior of reinforced soil.

**REFERENCES**


