DESIGN OF EARTHEN BARRIERS USING FINITE DIFFERENCE METHOD

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ABSTRACT: The earthen barrier used in waste disposal facility separates the waste from the ground water system to minimise the migration of contaminants from the facility. A simple solution technique using finite difference method (FDM) and a computer program CONTAMINATE-1D-FM have been developed to analyse contaminant migration of finite mass of pollutant available in the landfill and transport into the ground water system. This paper presents the design charts for the determination of thickness of earthen barriers and/or service life of liner based on the allowable values of relative base concentration in the aquifer of chloride and sodium species for a set of parametric values simulating the field condition using the proposed solution technique.

INTRODUCTION
The contamination from toxic wastes, subjected to leaching, migrated from landfills, is definitely one of the most burning problems to the groundwater system. Providing an effective engineered barrier, which will separate the waste from ground water, can minimise the potential contamination. The constructed clay liner media or the natural clay deposit is often situated in a site underlain by a thin natural aquifer [1]. In this paper a landfill has been considered separated by a clay liner of finite thickness in between the landfill and a thin permeable aquifer stratum. Advection velocity, dispersion coefficient, sorption potential, retardation factor, porosity, equivalent height of leachate and layer thickness have been considered to study the various hydrogeological conditions of the landfill sites. The objective of this paper is to develop the design charts of liner for chloride and sodium migration through saturated soil barrier and corresponding to the three values of equivalent heights of leachates as 1.0, 2.0 and 5.0 m for a set of performance criteria.

GOVERNING MODEL
The expression of top surface concentration, \( c_t \) at any time, \( t \) after the instant of peak concentration reached [1] is as follows:

\[
c_t(t) = c_0 - \frac{1}{H_f} \int_0^t f_0(c, t) dt
\]

Where, mass flux \( f = \eta c - \eta D \frac{\partial c}{\partial z} \)

\( \eta \) is the effective porosity of the liner material; \( v \) is the average linearised seepage velocity; \( c \) is the concentration of the contaminant; \( D \) is the dispersion coefficient; \( H_f \) is the equivalent height of leachate; \( f_0(c, t) \) is the surface flux at \( z=0 \);

The partial differential equation that describes the one-dimensional advection dispersion reaction equation (ADRE) for transport of non-decaying contaminant species through a homogeneous clay deposit/liner medium along the vertical downward direction is mentioned in Eq. 3.

\[
R \frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial z^2} - v \frac{\partial c}{\partial z}
\]

Where, \( R = \text{Retardation Factor} = 1 + \frac{K_u}{\eta} \)

The expression of concentration in the base aquifer, \( c_b \) at any time, \( t \) after the instant of peak concentration reached [1] is as follows:

\[
c_b(t) = \int_0^t \left( \frac{f_b(c, t)}{\eta_b h_b} - \lambda_b c_b(t) \right) dt
\]

Where, \( \lambda_b = \frac{v_b}{\eta_b L} \)

Where, \( v_b \) is the velocity in the base permeable layer; \( \eta_b \) is the porosity of the base aquifer; \( L \) is the length of the landfill in the direction of groundwater flow in the underlying aquifer; \( f_b(c, t) \) is the flux into the base aquifer; \( h_b \) is the thickness of the base aquifer layer.

BOUNDARY CONDITIONS
The initial concentrations of contaminant, at the top surface level of the barrier media, \( c_t(0) \); in the pore fluid within the barrier media, \( c(z, 0) \); and in the ground water within the base aquifer, \( c_b(0) \); have been assumed to analyse the model using proposed finite difference scheme as follows:

\[
c_t(0) = c_0 \quad \text{at} \quad z = 0 \quad \text{and} \quad t = 0
\]

\[
c(z, t) = 0 \quad \text{at} \quad z > 0 \quad \text{and} \quad t = 0
\]

and

\[
c_b(t) = 0 \quad \text{at} \quad t = 0
\]

The other boundary conditions assumed to analyse the model using proposed solution technique are as follows:

\[
0 \leq c_t(t) \leq c_0 \quad \text{at} \quad t > 0
\]

\[
c(z, t) = 0 \quad \text{at} \quad z > H \quad \text{and} \quad t > 0
\]

\[
c_b(t) \leq c_t(t) \quad \text{at} \quad t > 0
\]
PROBLEM DEFINITION
The problem geometry of this present study is illustrated in Fig. 1. The clay liner system is assumed to be homogeneous saturated layer underlying a thin permeable stratum of aquifer. The length and width of the landfill are considerably large to the thickness of the clay liner and therefore, one dimensional flow of the governing equation has been applied.

SOLUTION TECHNIQUE
A simple method FDM has been chosen in this study to solve the 1D model of contaminant transport through fully saturated homogeneous media underlying an aquifer simulating the field conditions. In the present study, the models mentioned in the earlier section (Eqs. 1, 3 and 5), the value of the concentration is dependent on time as well as space. The domain has been split into regular rectangular grids or meshes of width k in the time t – direction and depth h in the space z- direction; and illustrated in Fig. 2.

\[ z = ih \text{ and } t = jk \] (13-14)

The pivotal values at the points of intersection (known as grid points or nodes) denoted by \( c[j][i] \). In the present study, solution of the partial differential equations has been done by using forward difference scheme for single order derivative of concentration with respect to time; and central difference scheme both for single and second order derivatives of concentration with respect to space direction for better computation from the point of view of accuracy and stability [2].

Top Surface Concentration, \( c_t \)
The finite difference scheme, to predict the top surface concentration, \( c_t \) interfacing the landfill and the barrier medium, has been presented as follows:

\[ c_t[j] = \left( \frac{1}{FM_1} \right) \left( FM_2 c_t[j-1] + FM_3 c[j-1][1] \right) \] (15)

Where, \( c_t[j] \) is top surface concentration at different instant of time, t using the index j.

Base Aquifer Concentration, \( c_b \)
The finite difference scheme to predict the aquifer base concentration, \( c_b \) interfacing the barrier medium and the underlying more permeable aquifer stratum is as follows:

\[ c_b[j] = \left( -\frac{1}{P} \right) \left( Qc[j-1][i-1] + Sc[j-1][i] + Tc[j-1][i+1] \right) \] (19)

Where, \( j \) and \( i \) are the indices used in the two dimensional concentration arrays along time and space directions respectively.

\[ P = \frac{R}{k} \quad Q = -\frac{D}{h^2} - \frac{v}{2h} \quad S = \frac{2D}{h^2} - \frac{R}{k} \quad \text{and} \quad T = \frac{v}{2h} - \frac{D}{h^2} \] (20-23)

A program has been developed based on the above mentioned finite difference formulation (Eq. 19). The fitting parameters P, Q, S and T in the Eq. 19 are used based on the retardation factor, R; dispersion coefficient, D; seepage velocity, v; porosity of the barrier media, \( \eta \); the grid size h and k. This will also help in executing the programs.

Concentration of Pore Fluid within Liner at Different Depths at Various Time, \( c[j][i] \)
Using the approximation, the finite difference scheme of the model (Eq. 3) has been presented as follows.

\[ c[j][i] = \left( -\frac{1}{P} \right) \left( Qc[j-1][i-1] + Sc[j-1][i] + Tc[j-1][i+1] \right) \] (19)

Where, \( j \) and \( i \) are the indices used in the two dimensional concentration arrays along time and space directions respectively.

\[ c[j][i] = \left( -\frac{1}{P} \right) \left( Qc[j-1][i-1] + Sc[j-1][i] + Tc[j-1][i+1] \right) \] (24)

Where, \( c_t[j] \) is the concentration of the contaminant species
in the base aquifer at different instant of time, \( t \) using the index \( j \).
\[
B_1 = h k \eta_p h R L + 2 h \eta_p h R + k \eta D
\]  
(25)
\[
B_2 = h k \eta_p h R L - 2 h \eta_p h R + k \eta D
\]  
(26)
\[
B_3 = -h k \eta v - k \eta D
\]  
(27)

\( B_1, B_2 \) and \( B_3 \) are the fitting parameters used in the finite difference scheme to predict the base concentration in the aquifer, \( c_b \) in the finite difference scheme (Eq. 24) based on thickness of the aquifer, \( h_b \), length of landfill, \( L \); porosity of the aquifer stratum, \( \eta_b \); base velocity in the aquifer layer, \( v_b \); dispersion coefficient, \( D \); retardation factor, \( R \); and the grid size \( h \) and \( k \).

VALIDATION OF THE SOLUTION TECHNIQUE

The computer program CONTAMINATE-1D-FM has been used to solve the 1D models (Eqs. 1, 3 and 5) to study the response of contaminant migration and design of liners. An attempt has been made to compare the result of this study with the existing analytical solution of Booker-Rowe Equation [3] for validation of the proposed solution technique. The analytical solution for the concentration, \( c(z, t) \) at depth, \( z \) and time, \( t \) for the barrier having a diminishing surface concentration to take into account the advection and dispersion in the vertical direction for an infinitely deep deposit with a finite mass of contaminant in the source is as follows:
\[
c(z, t) = \left( \frac{c_0}{b - d} \right) \left[ \exp(ab - b^2t) \left( f(b, t) - df(b, t) \right) \right]
\]  
(28)

Fig. 3. Comparison of exit concentration-existing analytical solution and proposed solution techniques for varying top surface concentration

Where,  
\[
f(b, t) = \exp(ab + b^2t) \text{erfc}\left(\frac{a}{2\sqrt{t}}\right); \]
(29) 
\[
f(d, t) = \exp(ad + d^2t) \text{erfc}\left(\frac{a}{2\sqrt{t}}\right); \]
(30)
\[
a = \left(\frac{\eta + \rho K_d}{\eta D}\right)^{\frac{1}{2}}; \quad b = \left(\frac{\eta}{4D(\eta + \rho K_d)}\right)^{\frac{1}{2}} \quad \text{and}
\]

\[
d = \frac{\eta D}{H_f} \left(\frac{\eta + \rho K_d}{\eta D}\right)^{\frac{1}{2}} - b
\]  
(31-33)

In order to validate the proposed solution technique applying FDM, the typical values of the contaminant migration parameters \( H_f, \eta, D, \) and \( v \) used in this study are 1.00 m, 0.40, 0.001 m²/a and 0.005 m/a respectively. The comparative study has been made for the values of percentage of relative exit concentration in the liner at different thickness of liner of zero sorption potential after the periods of 20, 40 and 60 years. It is revealed from Fig. 3 that the results of this present study are very close to the existing analytical solutions.

RESULTS AND DISCUSSIONS

The numerical model presented in this paper has been simulated for the movement of chloride as conservative and sodium as non-conservative species through the saturated clay liner media underlying an aquifer. These two species have been selected due to commonly available species in the municipal waste landfill. The parametric values have been taken from previous research work and presented in Table 1. The design charts have been developed to facilitate the field engineers.

Table 1 Range of Parametric values studied

<table>
<thead>
<tr>
<th>Sl. No</th>
<th>Parameters studied/analysed values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Length of the landfill ( L ) (m)</td>
</tr>
<tr>
<td>2</td>
<td>Equivalent height of leachate, ( H_e ) (m)</td>
</tr>
<tr>
<td>3</td>
<td>Thickness of clay liner ( H ) (m)</td>
</tr>
<tr>
<td>4</td>
<td>Porosity of clay liner ( \eta ) (-)</td>
</tr>
<tr>
<td>5</td>
<td>Vertical downward advection velocity, ( v_a ) (m/a)</td>
</tr>
<tr>
<td>6</td>
<td>Dispersion coefficient ( D ) (m²/a)</td>
</tr>
<tr>
<td>7</td>
<td>Sorption potential ( \rho K_d ) (-): ( a ) Chloride, 0.00, ( b ) Sodium, 0.18</td>
</tr>
<tr>
<td>8</td>
<td>Thickness of permeable aquifer stratum, ( h_b ) (m)</td>
</tr>
<tr>
<td>9</td>
<td>Porosity of aquifer layer ( \eta_b ) (-)</td>
</tr>
<tr>
<td>10</td>
<td>Horizontal base velocity ( v_b ) (m/a)</td>
</tr>
</tbody>
</table>

Design Charts

This section presents the design charts for the determination of thickness of earthen barriers and/ or service life of liner based on the allowable values of relative base concentration in the aquifer for a set of parametric values, mentioned in Table 1, simulating the field condition based on the numerical model presented in earlier section using the proposed solution technique.
Attempt has been made to determine the liner thickness and service life of liner for conservative chloride species for 5% and 25% of relative base concentration ratio, $c_b/c_0$ and reactive contaminant species, sodium for 1% and 5% of relative base concentration ratio, $c_b/c_0$. Three values of equivalent heights of leachate, 1.0, 2.0 and 5.0 m have been considered to simulate the different hydrogeological conditions of the landfill sites and presented in Figs. 4, 5 and 6 respectively. The trends of the set of curves in the design charts are similar to those developed by Mathur and Jayawardena [4] using one-dimensional model. In the similar way design charts of clay liner may be developed for various landfill sites.

Fig. 4. Design chart of liner for chloride and sodium migration at $H_f = 1.0$ m

Fig. 5. Design chart of liner for chloride and sodium migration at $H_f = 2.0$ m

The required thickness of soil liner can be optimised from the design charts easily, corresponding to the set of performance criteria, service life and the allowable value in the base concentration ratio.

Fig. 6. Design chart of liner for chloride and sodium migration at $H_f = 5.0$ m

CONCLUSIONS

Based on the analysis and discussion presented herein the following conclusions may be drawn:

1. The values of equivalent height of leachate, $H_f$ can be used to take into account the finite mass of contaminant within a landfill at the top of the barrier simulating different field conditions.
2. The developed solution technique using FDM and the computer program CONTAMINATE-1D-FM have sufficient potential to analyse wide variety of contaminant migration in various hydrogeological conditions of the landfill site.
3. The design charts of liner are very useful to determine thickness or service life of earthen barriers.

REFERENCES