FOUNDATION SETTLEMENT IN SOILS SUBJECTED TO HIGH TEMPERATURE GRADIENT

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ABSTRACT: Foundation structures supporting nuclear, thermal power plants etc. have to be designed for extreme service conditions including uncontrolled thermal situations. In incidences where submerged soil under foundations is subjected to a high temperature gradient caused by failure of cooling systems, the pore water under the ground water table vapourises and escapes resulting in lowering of water table leading to settlement. As such, foundation design methodologies of above structures must give due consideration to settlements caused in the above scenario. Numerical approaches are complicated and require sophisticated softwares and advanced computers. This paper proposes a systematic theoretical approach to calculate heat transfer, temperature distribution in soil, vaporization of pore water from the ground water table and prediction of settlement caused by this process.

INTRODUCTION
The rapid demand for infrastructure has pressed geotechnical engineers to provide suitable foundation design for structures catering to unprecedented service conditions. Structures like thermal and nuclear power plants have to be designed not only for the routine service environment, but also for emergency conditions bordering on the possible breakdown situations. This is amply demonstrated by the recent uncontrolled rise of temperature in the reactor core of Fukushima Dai-ichi nuclear power plant in the aftermath of the tsunami. Such incidences underline the need for proper evaluation of extreme situations and design provision to ensure safety against the probable failures. This requires contribution by all the designers looking into the different aspects of the proposed structure. Consequently, it is the responsibility of Geotechnical engineers to provide foundation design to ensure that the foundations of such important structures continue to transmit loads without distresses like shear failures and settlements.

DEFINITION OF THE PROBLEM AND APPROACH TO ITS SOLUTION
When thermal and nuclear power plants situated in areas of high water tables have a situation of uncontrolled temperature rise due to lack cooling, the heat gets transmitted by conduction to the lower layers of soil. As the temperatures transmitted below the foundation level will decrease with depth, it is obvious that all the water in the soil will evaporate up to the depth where the temperature reaches 100 °C. Such a lowering of water table will result in settlement.

The intention here is to use heat equations with the properties of soil to find out the depth at which the temperature reaches 100 °C. Once this is calculated, using the computations for settlement, one can find the expected settlement under the foundations. The method proposed here will help the designers to take into consideration the settlements expected in extreme situations. This will help in ensuring safety against settlement failures under severe operation conditions. The method proposed here is of an elementary nature which can also be used as a check for the advanced analyses carried out using high end softwares.

COMPUTATION OF TEMPERATURE VARIATION
In order to compute and obtain the variation of temperature with depth, the soil through which the heat transmission takes place is considered as a homogeneous isotropic semi-infinite medium. Heat flow through this medium is computed and the temperature variation with respect to distance from the surface of the body for the given properties of soil for the input thermal conditions is obtained.

In the approach to the solution considering this as a semi-infinite solid problem, it is assumed that the heat convected or conducted to the surface of soil from the foundation is equal to the heat conducted into the surface. Since we are considering the steady state, it is also assumed that the losses during heat transfer have also reached a steady rate.

We consider that the semi-infinite solid is at a uniform initial temperature $T_i$ at time $t=0$. Then the surface of the solid at depth $x=0$ is exposed to convection or conduction by another solid (foundation) in continuous contact at a constant temperature $T_\infty$.

This problem can be formulated as a partial differential equation, which can be solved analytically for the transient temperature distribution at any depth and time, $T(x,t)$. This is expressed in Eq. 1 for the applicable boundary condition.
where $q_0$=heat conducted into the semi-infinite body from surface, $h$=heat transfer coefficient, $A$=contact area, $T$=temperature, $k$=thermal conductivity.[1,2]

Fig. 1 shows the diagrammatic representation of the medium adopted for the study and the boundary conditions applied to obtain a solution.

The solution of Eq. 1 for non-dimensionalised temperature $\theta(x, t)$ is given in Eq. 2

\[ 1 - \theta(x, t) = 1 - \frac{T(x, t) - T_\infty}{T_i - T_\infty} = \frac{T(x, t) - T_i}{T_i - T_\infty} \quad (2) \]

The initial temperature $T_i$ is suddenly changed to a temperature $T_\infty$ at the surface and maintained so. The expression for temperature distribution in the solid can be written as a function of time. This temperature distribution may be subsequently used to calculate heat flow at any depth $x$ in the solid as a function of time. The differential equation for temperature distribution $T(x, t)$ is indicated in Eq. 3,4.[3,4]

\[ \frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha^2} \frac{\partial T}{\partial t} \quad (3) \]

\[ \alpha^2 = k/C \rho \quad (4) \]

where $C$=specific heat, $\rho$=density of soil.

The boundary and initial conditions are shown in Eqs.5,6.

\[ T(x, 0) = T_i \quad (5) \]

\[ T(0, t) = T_\infty \quad \text{for } t > 0 \quad (6) \]

The Eq. 3 may be solved by Laplace transform technique. The solution is given by Eq. 7.

\[ \frac{T(x, t) - T_\infty}{T_i - T_\infty} = \text{erf} \left( \frac{x}{2\sqrt{\alpha t}} \right) = \text{erf} (\eta) \quad (7) \]

where the RHS is known as Gauss error function in the variable $\eta$ defined in Eq. 8. [2]

\[ \text{erf} \left( \frac{x}{2\sqrt{\alpha t}} \right) = \frac{2}{\sqrt{\Pi}} \int^{x/2\sqrt{\alpha t}}_{-\infty} e^{-\eta^2} d\eta \quad (8) \]

The plot of non-dimensionalised temperature with the error function is shown in Fig. 2. This relationship has been fitted and is given in Eq. 9.

\[ \text{erf} (\eta) = 0.087 \eta^3 - 0.561 \eta^2 + 1.314 \eta \quad (9) \]

Fig 2 Graph showing typical values of error function

With the help of Fig. 2, it is possible to easily obtain the temperature at any depth $x$ in the semi-infinite medium.

SETTLEMENT OF SOIL DUE TO VAPOURISATION OF PORE WATER AND SUBSEQUENT LOWERING OF GROUND WATER TABLE

When the temperature of the soil below the foundation reaches the boiling point (i.e. 100 °C) due to the heat transfer from the surface, pore water changes into vapour form and escapes through air voids. This results in the lowering of ground water table below the foundation up to the depth where the temperature is equal to or more than the boiling temperature.

The drawdown of water table due to vapourisation of water above it increases the effective stress on a compressible soil
Foundation settlement in soils subjected to high temperature gradient

...layer by changing the soil stress state from submerged and buoyant to dry conditions. The stress taken by the water in the pores is transferred to the soil grains as water escapes in vapour form. Due to the accompanying volume change, settlement in occurs in soil. [5,6]

The effective stress prior to the lowering of water table can be calculated from the Eq. 10.

$$\sigma_s = \sigma_s' - u_o$$

(10)

where $\sigma_s$ is the total stress, $\sigma_s'$ is the effective stress and $u_o$ the pore water pressure before drawdown of water table.

Similarly the effective stress after the lowering of water table by change in height of an amount $\Delta H$ can be calculated from the Eq. 11, taking $\gamma_w$ as the unit weight of water

$$\sigma_s = \sigma_s' - (u_o - \gamma_w \Delta H)$$

(11)

Considering the change in the unit weight of the zone subjected to dewatering on account of vapourisation, the amount of increase in the effective stress at the draw down level of water table is given by Eq. 12.

$$\sigma_s' - \sigma_s = \Delta \sigma = \Delta H \cdot \gamma_w - \Delta H (\gamma_{sw} - \gamma_d)$$

(12)

where $\gamma_{sw}$, $\gamma_d$ are the saturated and dry unit weights of soil.

The settlement of soil due to the increase in effective stress due to draw down of water table can be calculated using the Eq. 13 or Eq. 14.

$$S_c = m_v \cdot H_o \cdot \Delta \sigma$$

(13)

$$S_c = C_c \cdot H_o / (1 + e_o) \log (\sigma_s' + \Delta \sigma') / \sigma_s'$$

(14)

Where $m_v$=coefficient of volume compressibility of soil, $C_c$=compression index, $e_o$=the initial void ratio, $H_o$=initial thickness of soil layer and $\Delta \sigma'$=increase in effective stress at the centre of the soil layer due to the lowering of water table.

EXAMPLE CALCULATION FOR PREDICTION OF SETTLEMENT

Consider a foundation for a nuclear power plant resting on a pervious sand stratum (S1) of 20 m thickness. Water table is near the ground level. Beneath the top layer of soil there is a cohesive soil layer (S2) of thickness 20 m. The initial uniform temperature of foundation soil is 20 °C. Due to the sudden uncontrolled heating from the reaction chamber of the power plant let the temperature just below the foundation be increased to 1000 °C. For numerical calculation, the soil properties are assumed as indicated in Table 1.

Table 1. Soil properties used for the sample calculation

<table>
<thead>
<tr>
<th>Property of soils</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saturated unit weight (S1)</td>
<td>20.4 kN/m³</td>
</tr>
<tr>
<td>Dry unit weight (S1)</td>
<td>16.5 kN/m³</td>
</tr>
<tr>
<td>Submerged unit weight (S1)</td>
<td>10.6 kN/m³</td>
</tr>
<tr>
<td>Thermal Conductivity (k) (S1)</td>
<td>0.3 Wm⁻² °C⁻¹</td>
</tr>
<tr>
<td>Specific heat (C) (S1)</td>
<td>0.8 Jkg⁻¹ °C⁻¹ x10³</td>
</tr>
<tr>
<td>Coeff. of vol. compr. (m_v) (S2)</td>
<td>1.02 x10⁻⁵ m²/kN</td>
</tr>
</tbody>
</table>

Step 1

We rewrite Eq. 7 in the form shown in Eq. 15.

$$T(x, t) = T_{\infty} + (T_i - T_{\infty}) \erf \frac{x}{2\sqrt{\alpha t}}$$

(15)

Assign the given data, $T_i$=1000 °C, $T_f$=20 °C, $H_o$=25 m and calculate $\alpha$ for soil properties given in Table 1. Taking $k$=0.3 Wm⁻² °C⁻¹; $C$=0.8 Jkg⁻¹ °C⁻¹ x10³; $p$=1.06 kgm⁻³ x10⁵ and substituting in Eq. 4, we get $\alpha=0.000595$.

In order to obtain the error function, we put a large value for time t as we have assumed and are interested in the steady state conditions. On substitution of the data in the example, Eq. 15 can be written as in Eq. 16. The value for error function is taken from Eq. 9.

$$T(x, t) = 1000 + (20 - 1000) \erf \frac{x}{2\sqrt{0.000595 t}}$$

(16)

Substituting for $x=15$ m and a large value of $t=100000$ s, we get the corresponding temperature $T(x, t) = 189$ °C.

Step 2

Using Eq. 16, for a given large t, the temperature T is computed plotted with against depth in Fig. 3 and tabulated in table 2. Either using the Eq. 16 or from the Fig. 3, the depth corresponding to the temperature of 100 °C is found out. This is the depth above which water has vapourised. Soil from the surface up to this depth will get dried up resulting in settlement, which will be calculated in the next step. From Fig. 3, in the present case, the temperature is found to become 100 °C at a depth of 18.47 m.

Step 3

The settlement of cohesive soil layer (S2) due to the lowering of water table in layer (S1) is obtained substituting the properties of soil (S2) in Eq. 12 as shown below. Here the settlement in the sand layer (S1) is not considered as this value is negligible compared with the larger settlement in the compressible cohesive layer.

$$S_c = m_v \cdot H_o \cdot \Delta \sigma$$
The increase in effective stress is found out by substituting the properties of soil (S1) in Eq. 11

\[
\Delta \sigma = \Delta H \gamma_w - \Delta H (\gamma_{sat} - \gamma_d) \\
= 20 \times 9.80 - 20 \times (20.4 - 16.5) \\
= 118 \text{ kN/m}^2
\]

Sc = 1.02 \times 10^{-3} \times 20 \times 118 \\
= 2.41 \text{ m}

Therefore, considering a thickness of about 20 m of soil, for the soil properties taken into account, the estimated settlement due to lowering of water table on account of vaporisation of pore water due to uncontrolled heat transmission beneath foundation is estimated to be of the order of 2.41 m, which is a dramatic settlement that should be anticipated in the foundation design stage and measures adopted to mitigate the effects of such a settlement.

CONCLUSIONS

1) Steady heat transfer through soil can lead to settlement when lowering of water table takes place. Lowering of water table takes place up to the depth where the temperature of soil is above boiling point. There is a need for a simple step by step approach to estimate temperature variation in soil and the consequent settlement in the design of foundations for structures such as thermal and nuclear power plants.

2) Considering the heat transmission as taking place in a homogeneous isotropic semi-infinite medium, the problem has been formulated as a partial differential equation, and solved analytically for the transient temperature distribution at any depth and time, \( T(x,t) \).

3) The solution proposed here helps to predict the variation of temperature with time and depth considering the soil properties such as thermal conductivity, specific heat, and density of soil.

4) On ascertaining the depth up to which water evaporates, settlement can be computed for the depth of soil which was initially submerged but has now become dry.

5) Possible settlement in extreme temperature conditions can be predicted under the stated assumptions by hand calculations using the method given here as a check to sophisticated computer based analyses.

REFERENCES