ANALYSIS OF RETAINING WALL WITH PRESSURE RELIEF SHELF BY COULOMB’S METHOD

R. D. Padhye, Lecturer, WCE Sangli (MS), e mail: rajeshpadhye1@rediffmail.com
P. B. Ullagaddi, Professor, SGGSIT Nanded (MS), email: pbu@rediffmail.com

ABSTRACT: A cantilever retaining wall with pressure relief shelf may be one of the special types of retaining walls. The concept of providing pressure relief shelf towards the back fill side of retaining wall reduces the total earth pressure on the wall which results in reducing the sections of wall and ultimately to get an economical wall. The paper presents the theoretical analysis of cantilever retaining wall with pressure relief shelf by using Coulomb’s method. The reduced total active earth pressure due to provision of shelf has been calculated from the stability analysis of wedges formulated by Coulomb’s theory.

INTRODUCTION

A retaining wall is a structure designed to sustain the lateral pressure of earth behind it. It retains a steep faced slope of an earth mass against rupture of slopes in cuts and fills and against sliding down. The lateral force acting between the retaining structure and the retained earth mass is termed as lateral earth pressure.

Retaining wall with pressure relief shelves is one of the special types of retaining wall. High reinforced concrete retaining walls may be used economically by providing relief shelves on the back fill side of the wall. Such walls may be termed as the “Retaining Wall with Pressure Relief shelf”

According to Jumikis, the provision of one or more relief shelves and extending them to the rupture surface, can considerably increase the stability of retaining wall. The relief shelves have an advantage of decreasing the overall lateral earth pressure on the wall and increasing the stability of the structure. This results in an economical design because less material goes into the wall as compared to massive structure of cantilever or even counterfort retaining walls without shelves.

Raychaudhuri presented Coulomb’s theory for earth pressure computation, analysis and design of this type of wall for cohesionless soil. He proposed the magnitude of reduction in total active earth pressure and its distribution due to the provision of a relief shelf in a retaining wall. He presented the reduction factors in the form of ready to use charts for various locations and widths of relief shelf. Author have experimentally proved that the total active earth pressure on a retaining wall with relief shelf is lower in magnitude than that of a conventional type and the keying at the base may not be necessary to prevent sliding in certain cases. He performed the experimentation work on sliding and overturning of this type of wall.

Phatak gave the theoretical concept for evaluating the earth pressure due to the effect of relief shelves by using the Rankine’s theory.

COULOMB’S THEORY

The Coulomb’s theory is conveniently adopted when the plane of failure extending diagonally upward and backward through the backfill. The sliding wedge is a triangular mass of soil between this plane of failure and the back face of retaining wall. The soil within the sliding wedge would slump down when the retaining wall is suddenly removed. If a plane of failure makes an angle $\phi$ with the horizontal, the forces acting on the sliding wedge are as shown in figure 1.

These forces consist of weight of the soil within the wedge ($W$) which acts through the centroid of the triangle, a thrust normal to the plane of failure ($N$) which exerts by the soil to the right of the failure plane. $N = N \tan \phi$ will be at the limit of equilibrium. These forces must be balanced by the
thrust P which is assumed to act horizontally and to be concurrent with W, N and S. The equal and opposite reaction to P is the lateral force to withstand which the wall is to be designed.

The forces N and S may be replaced by the resultant R to derive the value of P. S acts along a line making the angle \( \phi \) with the normal to the failure plane. Since W, P and R are three concurrent forces which are in equilibrium, when the failure is about to take place along the failure plane, they may be represented by the triangle of forces in figure 1.

In this triangle

\[
P = W \tan (\rho - \phi)
\]

\[
W = \frac{1}{2} \gamma H^2 \cot^2 \rho
\]

\[
P = \frac{1}{2} \gamma H^2 \cot^3 \rho \tan (\rho - \phi)
\]

But as \( \gamma, H \) and \( \phi \) are constant for any particular wall and soil, this equation indicates that, P varies with the angle \( \rho \).

To obtain the maximum value of P, \( \frac{dP}{d\rho} \) is to be equated to zero to solve for \( \rho \). and solving for value of \( \rho \) it can be shown that P is maximum when \( \rho \) is equal to \( (45 + \phi/2) \).

Substituting back we get

\[
P = \frac{1}{2} \gamma H^2 \tan (45^\circ - \phi/2)
\]

\[
P = \frac{1}{2} \gamma H^2 \frac{1 - \sin \phi}{1 + \sin \phi}
\]

\[
P = \frac{1}{2} \gamma H^2 K_a
\]

Where,

\[
K_a = \frac{1 - \sin \phi}{1 + \sin \phi}
\]

The above equation is nothing but the Rankin’s formula for a level backfill.

**APPLICATION OF COULOMB’S THEORY TO RETAINING WALL WITH SHELF**

The effect of relief shelf can be considered by deducting the weight of soil above the relief shelf from failure wedge. For non-cohesive soils the active earth pressure on a retaining wall can be computed by considering the stabilities of different wedges of soil mass. It attains a maximum value when the rupture surface makes an angle of \( (45^\circ + \phi/2) \) with the horizontal, where \( \phi \) is the angle of internal friction of the non-cohesive soil.

Figure 2 shows the cross section of a retaining wall with one horizontal relief shelf of width \( b \) and thickness \( t \) at a height \( (H - h) \) from the base.

When \( b = (H - T - h) \tan (45^\circ - \phi/2) \), the rupture plane originating at the intersection of the base and stem on the backfill side meets the horizontal shelf. Figure also shows the changed pressure distribution diagram due to provision of single shelf. According to Jumikis the earth pressure distribution diagram below the shelf would be as shown in figure, as if a free surface existed at the shelf level.

The reduced total active earth pressure due to provision of a shelf has been calculated from the stability analysis of wedges formulated by Coulomb and has been expressed for easy computation as a fraction of the total active earth pressure for a retaining wall without shelf. This fraction which represents the reduction factor when multiplied by the total earth pressure for conventional retaining walls gives the magnitude of total earth pressure with relief shelf.

If \( b \) is greater than \( (H - T - h) \tan (45^\circ - \phi/2) \), the rupture plane which gives the maximum value of lateral earth pressure, i.e. the plane inclined at \( (45^\circ - \phi/2) \) with the horizontal, cannot develop, as it has to go through the shelf. Though the total active earth pressure, when a rupture plane makes an angle other than \( (45^\circ - \phi/2) \) with horizontal, is less than the maximum, its magnitude can be larger than the total earth pressure obtained from figure. Hence, the earth pressure distribution diagram may follow the pattern as shown in figure when a shelf is provided to a retaining wall.

The reduced total active earth pressure at any level can be obtained by stability analysis of the wedge, assuming that by providing a horizontal shelf, the weight of the earth over the shelf is taken up by the shelf and the weight of this soil mass is not effective to cause sliding.

As already mentioned, the total active earth pressure at any level may be obtained for different values of \( b \) and \( h \), and hence Coulomb’s theory was adopted as follows.

The soil wedge behind a retaining wall is subjected to the three forces, viz, its weight \( W \), reaction \( R \) on the rupture
surface and reaction from the wall induced by the active earth pressure \( P \). The instant, the equilibrium of these forces is destroyed, failure occurs. For the equilibrium condition, it is necessary that these forces meet at a common point. From the method of force triangle it can be shown that –

\[
P = W \cdot \frac{\sin(\phi - \theta)}{\sin(\phi + \rho - \theta)}
\]

Where,

\( \rho \) is the angle of the rupture plane with the horizontal
\( \phi \) is the angle of internal friction of soil.
\( \alpha \) is the angle of inclination of the back face of wall with the vertical

\[
\phi = (90^\circ - \alpha - \theta)
\]

Where,

\( \theta \) is the angle of friction between backfill and wall.
The above equation is valid when the angle of slope of backfill is zero. When the back face of wall is vertical, which is commonly met in practice, \( \alpha = 0 \). Then the equation becomes,

\[
P = W \cdot \frac{\sin(\rho - \phi)}{\cos(\rho - \phi + \theta)}
\]

If \( \gamma \) is the dry density of soil and \( H \) is the height of the wall, \( W \) becomes equal to \( \gamma H^2 \cot \rho \), per unit width of wall. When a shelf of width \( b \) is provided at a particular height \( h \) from the top of the wall, the weight of soil above the shelf that does not contribute to the slide is \( \gamma b h \) which can also be written as –

\[
\frac{b}{H} \cdot \frac{h}{H} \cdot H^2
\]

Defining width factor \( \beta \) as equal to \( \frac{b}{H} \), and location factor \( \delta \) equal to \( \frac{h}{H} \), the weight of the soil above shelf becomes \( \gamma \beta \delta H^2 \). Hence the weight of the wedge that causes slide becomes –

\[
\left( \frac{\gamma}{2} \cot \rho - \gamma \beta \delta \right) H^2
\]

The value of reduced active earth pressure \( Pr \) is calculated by the equation –

\[
Pr = \frac{\left( \frac{\gamma}{2} \cot \rho - \gamma \beta \delta \right) \sin(\phi - \theta) H^2}{\cos(\theta - \rho + \phi)}
\]

This reduced active earth pressure will act at the changed position of the centre of gravity of the failure wedge due to provision of shelf. The required reduction factor would be \( P_r/P \).

It may be noted that for a retaining wall of height \( H \), the reduced earth pressure is a function of many terms out of which \( \rho \) is the only unknown. Though the maximum value of \( P_r \) can be obtained by differentiation, it has been determined by numerical methods. The reduction factor \( P_r/P \) may be plotted against \( \beta \), for various values of \( \delta \).

CONCLUSIONS

The coulomb’s theory can be conveniently adopted for the analysis of earth pressure behind retaining wall with relief shelf. The reduced earth pressure may be calculated by simply multiplying the value of \( P \) by the reduction factor. The active earth pressure and lever arm are substantially reduced due to provision of shelf and thereby achieves a considerable reduction in the moment about the base.

REFERENCES