ABSTRACT: This paper presents the behavior of rigid retaining wall backfilled with cohesionless soil under static and dynamic (pseudo-static and pseudo-dynamic) passive earth pressures. The unit weight is assumed to be constant throughout the soil mass. In the static analysis, the Coulomb’s theory and Kötter’s equation have been adopted for the determination of passive earth pressures under varying height of wall. These results are plotted and found to be matching well. In the dynamic analysis, Kötter’s equation has been used. The coefficient of seismic passive earth pressure is computed for different soil and wall properties. For the same cases, the seismic passive earth pressure with varying height of wall are plotted and reported in the present paper.

INTRODUCTION
Estimation of the static and dynamic earth pressure is an important topic of research for the safe design of retaining wall in the seismic zone. Pioneering works by Okabe [2] and Mononobe and Matsuo [3], commonly known as Mononobe-Okabe method based on the pseudo-static approach, gives the linear earth pressure distribution in a very approximate way irrespective of static and seismic conditions. Dewaikar and Halkude [4] have proposed a pseudo-static numerical analysis of seismic passive thrust on retaining wall, using Kötter’s equation. The analytical method based on pseudo-dynamic approach as given by Steedman and Zeng [5] and modified by Choudhury and Nimbalkar [6] is used for the present analysis for calculation of seismic passive earth pressure. In this paper, a complete analytical study describing the behaviour of static and dynamic earth pressure distribution for different soil and wall properties for passive conditions is presented.

ANALYSIS OF STATIC EARTH PRESSURE
Various passive earth pressures were calculated for a retaining wall which retains dry cohesionless backfill.

The passive earth pressure for variations of backfill angle ($\beta = 0^\circ$, $5^\circ$, $10^\circ$ and $15^\circ$) and angle of internal friction ($\phi = 20^\circ$, $30^\circ$ and $40^\circ$) are computed using Coulomb’s theory as well as Kötter’s equation. The unit dry weight of soil was considered $15 \text{ kN/m}^2$ and $\Delta = 2/3 \phi$. Fig. 1 shows a retaining wall with an inclined back face and a sloping dry granular backfill. The backfill consists of homogeneous, elastic and isotropic cohesionless soil. The passive thrust, $P_p$, is determined as:

$$P_p = \frac{1}{2} \gamma H^2 K_p$$

where,

$$K_p = \frac{H^2 \sin(\phi) \sin(\alpha + \beta) \sin(\alpha + \phi)}{(\sin \phi)^2 \sin(\beta - \phi) \sin(180 - \gamma - \phi + \phi)}$$

ANALYSIS OF PSEUDO-STATIC EARTH PRESSURE
Analysis of pseudo-static passive earth pressure is done by Kötter’s equation, which is given by:

$$\frac{dp}{ds} + 2p \tan \phi \frac{d\alpha}{ds} = \gamma \sin(\alpha - \phi)$$

For pseudo-dynamic case, the variations of backfill angle ($\beta = 0^\circ$, $5^\circ$, $10^\circ$ and $15^\circ$), horizontal acceleration coefficient ($k_h = 0.1$, $0.2$ and $0.3$), vertical acceleration coefficient ($k_v = 0.1$, $0.2$ and $0.3$), and angle of internal friction ($\phi = 20^\circ$, $30^\circ$ and $40^\circ$) are computed. The unit dry weight of soil was considered $15 \text{ kN/m}^2$ and $\Delta = 2/3 \phi$. If $k_h = k_v = 0$, then the Kötter’s equation becomes a static case, which is used for computing the static passive earth pressure.

The free body diagram of this wedge is shown in Fig. 2 with the orientation of the forces. The forces that are involved are:

- the active thrust $P_a$weight of ABC failure wedge $W$, inertia forces $k_v W$ and $k_h W$ (where, $k_v$ = vertical acceleration coefficient and $k_h$ = horizontal acceleration coefficient) in vertical and horizontal directions respectively, and soil
reaction \( R \) on failure plane \( AB \). For a plane surface, \( da/ds = 0 \) and Eq. (2) takes the following form,

\[
\frac{dp}{ds} = \gamma \sin(\alpha - \phi)
\]

(3)

Fig. 2 Free body diagram of failure wedge ABC with orientation of the forces

The passive thrust, \( Pp \) is determined when the horizontal and vertical forces are in equilibrium. The horizontal force equilibrium is:

\[
P_p = \frac{R \sin(\alpha + \phi) - K_h \cdot W}{\sin(\theta + \delta)}
\]

(4)

and, the vertical force equilibrium.

\[
P_p = \frac{(1 - K_v) \cdot W - R \cos(\alpha + \phi)}{\cos(\theta + \delta)}
\]

(5)

where,

\[
R = \frac{1}{2} \cdot \gamma \cdot H^2 \cdot \sin^2(\theta + \delta) \cdot \sin(\alpha + \phi) \cdot \tan(\alpha - \phi) \cdot \sin^2 \theta
\]

(6)

The equations (4) and (5) will yield the same unique value of \( P_p \) only when the equilibrium conditions correspond to those at failure, which are uniquely defined by a characteristic value of \( \alpha \) which is determined by trial and error procedure. Thus, in this method of analysis, failure plane is identified in such a manner that the force equilibrium of failure wedge ABC as shown in Fig. 2 is satisfied. This approach is different from Coulomb’s analysis in which \( P_p \) is obtained from the consideration of its maximum value, i.e. \( \alpha \) is identified in such manner that maximum value of \( P_p \) is obtained from force equilibrium considerations. Table 1 shows the critical soil wedge angle for the variations.

### ANALYSIS OF PSEUDO-DYNAMIC EARTH PRESSURE

The pseudo-dynamic earth pressures for passive case are calculated for a retaining wall which retains dry cohesionless backfill by Köttér’s theory. The pseudo-dynamic analysis considers finite shear wave and primary wave velocities. Consider the fixed base vertical cantilever wall, \( AB \) of height \( H \) as shown in Fig. 4. The wall is supporting a cohesionless backfill material with horizontal ground. Both the shear wave velocity and primary wave is assumed to act within the soil media due to earthquake loading.

The different parameters used are as follows: wall friction angle \( \delta \), soil friction angle \( \phi \), shear wave velocity \( V_s \), primary wave velocity \( V_p \), horizontal seismic acceleration \( k_h \cdot g \), vertical seismic acceleration \( k_v \cdot g \).

<table>
<thead>
<tr>
<th>( k_h )</th>
<th>( \beta )</th>
<th>( \phi = 20^\circ )</th>
<th>( \phi = 30^\circ )</th>
<th>( \phi = 40^\circ )</th>
</tr>
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<tr>
<td>0.1</td>
<td>5^\circ</td>
<td>28.215</td>
<td>20.060</td>
<td>12.017</td>
</tr>
<tr>
<td>0.2</td>
<td>5^\circ</td>
<td>31.957</td>
<td>23.084</td>
<td>14.630</td>
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<tr>
<td>0.3</td>
<td>5^\circ</td>
<td>35.157</td>
<td>25.839</td>
<td>17.102</td>
</tr>
<tr>
<td>0.1</td>
<td>10^\circ</td>
<td>35.157</td>
<td>25.839</td>
<td>17.102</td>
</tr>
<tr>
<td>0.2</td>
<td>10^\circ</td>
<td>37.775</td>
<td>27.378</td>
<td>17.781</td>
</tr>
<tr>
<td>0.3</td>
<td>10^\circ</td>
<td>39.554</td>
<td>29.154</td>
<td>18.900</td>
</tr>
</tbody>
</table>

The variations of horizontal acceleration coefficient \( k_h = 0.1, 0.2 \text{ and } 0.3 \), vertical acceleration coefficient \( k_v = 0.1, 0.2 \text{ and } 0.3 \), and angle of internal friction \( \phi = 20^\circ, 30^\circ \text{ and } 40^\circ \) are computed using Köttér’s theory. The unit dry weight of soil was considered 15 kN/m³ and \( \Delta = 2/3 \phi \). It is assumed that the shear modulus, \( G \) is constant with depth of retaining wall throughout the backfill.

The seismic inertia force in horizontal direction is expressed as,

\[
Q_h = \frac{\lambda}{2\pi} \cdot K_v \cdot H \cdot \cos2\eta \left( \frac{t}{T} - H \right) + \frac{\lambda}{2\pi} \cdot \sin2\eta \left( \frac{t}{T} - H \right) \cdot \sin2\eta \left( \frac{t}{T} \right)
\]

(7)

where \( \lambda = T V_s = \text{wave length of } s \text{-wave} \)

Similarly, vertical acceleration is expressed as,

\[
Q_v(\alpha, t) = \frac{\gamma}{2\pi \eta \tan(\phi)} \cdot \left[ \frac{\sin2\eta \left( \frac{t}{T} - H \right) - H \left( \frac{t}{T} - \sin2\eta \left( \frac{t}{T} \right) \right)}{\cos2\eta \left( \frac{t}{T} \right)} \right]
\]

(8)

where \( \eta = T V_p = \text{wave length of } p \text{-wave} \)
The seismic passive earth pressure coefficient, $K_{pe}$ is defined as, $K_{pe} = \frac{2 P_{pe}}{(\gamma H^2)}$

RESULTS AND DISCUSSIONS

Results are presented in graphical form for normalized Passive earth pressures along the normalized depth of the wall ($z/H$). Variations of Parameters considered are as follows: $\phi = 20^0, 30^0$ and $40^0$; $\delta/\phi = 2/3$; $k_h = k_v = 0.0, 0.1, 0.2, 0.3$.

Static Passive Earth Pressure

In this study, the static earth pressure in passive state are computed and compared by Coulomb’s theory and Kötter’s equation for different soil and wall properties. The passive earth pressure in both the cases was found to be identical. Fig. 5 shows the passive earth pressure distribution with depth for different values of soil friction angle, $\phi$ under static condition by Coulomb’s theory and Kötter’s equation (i.e. $k_h = k_v = 0$). It is clear that under static condition, the passive earth pressure distribution is identical as expected.

Pseudo-Static Passive Earth Pressure

Pseudo-static methods are applied to determine seismic passive earth pressure distribution using Kötter’s equation. Fig. 7 shows the pseudo-static passive earth pressure distribution with depth for different values of soil friction angle, $\phi$ under pseudo-static condition (i.e. $k_h = k_v = 0.1$).

From Fig. 7, it is seen that pseudo-static earth pressure shows significant increase with increase in angle of internal friction, $\phi$. For example, with $\delta = 2/3$, it is seen that $k_h = k_v = 0.1$; as $\phi$ increases from $20^0$ to $40^0$.

Results of the normalized pseudo-static passive earth pressure distribution with normalized depth for different values of soil friction angle, $\phi$ under seismic condition of $k_h = 0.1$ and $k_v = 0.1$, are shown in Fig. 8. It is clear that under pseudo-static condition, the passive earth pressure distribution is exactly linear.

Fig. 5 Variation of passive earth pressure with depth for different values of soil friction angle by Coulomb’s theory and Kötter’s equation

Fig. 6 shows the passive earth pressure distribution with normalized depth for different values of soil friction angle, $\phi$ under static condition (i.e. $k_h = k_v = 0$). It is clear that under static condition, the passive earth pressure distribution is exactly linear.

Fig. 8 Normalized pseudo-static passive earth pressure distribution with normalized depth for different values of soil friction angle

Fig. 9 Pseudo-static passive earth pressure distribution with depth under seismic condition of $k_h = k_v = 0, 0.1, 0.2, 0.3$ with $\phi = 20^0$
Figure 9 shows the pseudo-static passive earth pressure distribution with depth for soil friction angle, \( \varphi = 20^\circ \) under pseudo-static condition (i.e. \( k_h = k_v = 0, 0.1, 0.2, 0.3 \)). From Fig. 9, it is seen that as \( k_h \) increases, seismic passive earth pressure decreases, for example, with \( \delta = 2/3\varphi \) and \( \varphi = 20^\circ \), as \( k_h = k_v \) increases from 0.0 to 0.3, keeping all other parameters same.

**Pseudo-Dynamic Passive Earth Pressure**

Pseudo-dynamic methods are applied to determine seismic passive and passive earth pressure distribution using Kötter’s equation. Results of the pseudo-dynamic passive earth pressure distribution with depth for different values of soil friction angle, \( \varphi \) under seismic condition of \( k_h = k_v = 0.2 \), are shown in Fig. 10. From Fig. 10, it is seen that pseudo-dynamic earth pressure shows significant increase with increase in angle of internal friction \( \varphi \). For example, with \( \delta = 2/3\varphi \), \( k_h = k_v = 0.2 \), as \( \varphi \) increases from 20° to 40°.

**CONCLUSIONS**

In the present study, the static earth pressure in passive state are computed by Coulomb’s theory and then compared with Kötter’s equation. Comparison of these earth pressures is done which are found to be identical and give linear pressure distribution with depth.

Pseudo-static earth pressure varies according to seismic acceleration coefficient (\( k_h, k_v \)) showing linear pressure distribution along the depth of soil. Pseudo-dynamic earth pressure varies according to seismic acceleration coefficient (\( k_h, k_v \)) shear and primary waves propagating in the backfill behind the rigid retaining wall on the seismic earth pressures. It gives more realistic non-linear seismic passive earth pressure distribution behind the retaining wall as compared to the pseudo-static approach.

**REFERENCES**