STRESS DEPENDENT ACTIVE EARTH PRESSURES FOR HIGH RETAINING WALLS

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ABSTRACT: Rankine’s and Coulomb’s earth pressure theories, widely used to estimate the active earth pressure ignore stress-dependency on the mobilized angle of shearing resistance and assume that the frictional resistance is mobilised uniformly along the slip surface and soil-to-soil friction coefficient $f$ is equal to $\tan \phi$. For high retaining walls, influence of effective confining stress on the mobilized friction angle along the slip surface can be significant due to large variation of mean effective stress with depth. In this paper, active earth pressures against high retaining walls ($H=5\text{ m}, 10\text{ m}, 20\text{ m}, 30\text{ m and } 40\text{ m}$) are evaluated accounting for the stress-dependency of the friction angle on the mean effective stress.

INTRODUCTION

For the design of earth structures, Coulomb’s theory proposed in 1773 is widely used for calculating the earth pressures against retaining walls. Based on this theory, lateral thrust against the wall is estimated by static analysis of forces acting on a sliding wedge at incipient failure. By choosing a series of slip planes inclined at angle $\theta$, lateral thrust is calculated and the maximum value is taken as the critical value (active thrust) for retaining wall design (Fig. 1). In this analysis, the mobilized friction angle $\phi$ is assumed to be constant along the slip plane. However, since the early works of Taylor (1960) and Rowe (1962), many researchers have studied the effect of effective confining stress on the friction angle and the dilatancy of soil sample during shearing, and proposed relationships between the effective confining stress and the peak friction angle (Bolton 1986, Been and Jefferies 1985, Manzari and Dafalias 1997, Li and Dafalias 2000, Chakraborty and Salgado 2010). In this paper, effect of stress dependency is considered on the mobilized friction angle along the slip plane and the active earth pressures are calculated for high retaining walls up to 40 m, where the stress-dependency can be significant on the mobilized friction angle.

CONVENTIONAL METHOD

Conventional method for design of retaining wall assumes a constant friction angle $\phi$ to be mobilized along the slip plane in the determination of the total thrust on the wall. For example, the total lateral thrust (active condition) using Coulomb’s method is obtained as $P = \frac{1}{2}k_a^1H^2$, $k_a$ is the coefficient of earth pressure and is equal to $\frac{(1-\sin \phi)}{(1+\sin \phi)}$. The failure plane is found to be inclined at an angle $\theta_f = 45 + \phi/2$. The constant friction angle $\phi$ is typically obtained by conducting direct shears tests on the backfill material in the normal stress range of 50-200 kPa.

PROBLEM DEFINITION

To determine the active earth pressure for a wall of height $H$ with a level backfill with properties - unit weight $\gamma$, critical state friction angle $\phi_c$ and relative density $D_r$ by considering the effect of mean confining stress on the mobilized friction angle along the slip plane.

Fig. 1 Coulomb’s method for calculation of earth pressure for the case of zero wall friction: (a) slip plane assumed at an angle $\theta$, (b) forces acting on sliding wedge, (c) polygon of forces, and (d) determination of maximum value of $P$

PROPOSED METHOD

The Horizontal Slice Method proposed by Shahgoli et al. (2001) is modified and adopted to estimate the active earth pressure distribution with depth. In this method, slip plane is assumed and the sliding wedge is divided into a number of horizontal slices, 1 to $n$ (Fig. 2a). The forces acting on an $i^{th}$ slice are shown in Fig. 2b.

Considering the equilibrium of forces in the horizontal and vertical directions for slice $i$ give

\[ \sum F_y = 0 \Rightarrow V_{i+1} - V_i + S_i \sin \theta + N_i \cos \theta = 0 \]  
(1)

Where

\[ S_i = c_b + N_i \tan \phi \]  
(2)

Substituting Eq. (2) in Eq. (1) and solving for $N_i$, one gets

\[ N_i = \frac{V_i - V_{i+1} + W_i - c_b \sin \theta}{\tan \phi \sin \theta + \cos \theta} \]  
(3)
Fig. 2 Horizontal slice method: (a) backfill behind the wall height divided into \( n \) horizontal slices, and (b) forces on a slice

Considering equilibrium of forces in the x-direction for the whole wedge, one gets

\[
\sum F_x = 0 \Rightarrow P = \sum_{i=1}^{n} (N_i \sin \theta) - \sum_{i=1}^{n} (S_i \cos \theta) \tag{4}
\]

where \( V_i \) and \( V_{i+1} \) are the vertical interslice forces (obtained by integration of overburden pressures on horizontal borders), \( H_i \) and \( H_{i+1} \) are the horizontal interslice forces, \( W_i \) is the weight of the slice, \( N_i \) is the normal force at the base of the slice, \( S_i \) is shear strength of the backfill, \( c \) is the cohesion of the backfill, \( b_i \) is the base width of the slice along the slip plane, \( \phi \) is the mobilized friction angle of the backfill material.

**APPRAOCH**

The stress state of the soil element along the slip plane for each slice is determined assuming the horizontal and vertical planes to be the principal planes. For an assumed value of \( \phi \), mean effective confining stress \( \sigma_{mc} \) is obtained and the assumed value of \( \phi \) is compared with that obtained from Bolton’s relation for plane-strain condition [Eq. (5)], and the iterations are continued until the assumed \( \phi \) value converges with that of the calculated value.

\[
\phi = \phi + 5I_k \tag{5}
\]

where, \( I_k \) is given by

\[
I_k = I_0 \left[ Q - \ln \left( \frac{100 \sigma_{mc}}{\sigma_{p3}} \right) \right] - R_0 \tag{6}
\]

where, \( I_0 = D_k/100 = \) relative density ranging from 0 to 1; \( Q, R_0 \) = parameters that depend on intrinsic characters of sand (generally 10 and 1 respectively); \( \sigma_{p3} \) = reference stress = 100 kPa; \( \sigma_{mc} = \) mean effective stress at peak shear strength. \( \sigma_{mc} = 0.5(\alpha' + \sigma'_{3}) \) for plane-strain conditions, \( \sigma' \) and \( \sigma'_{3} \) are the major and minor principal stresses.

For each slice, \( N_i \) and \( S_i \) are then calculated using Eq. (2) and Eq. (3) and the total thrust is calculated using Eq. (4).

**COMPARISON OF CONVENTIONAL AND PROPOSED METHODS**

Guo (2008) performed direct shear tests on Ottawa sand and reported a friction angle of 38 degrees in the normal stress range 50-200 kPa at a relative density \( D_k=86\% \). Critical-state friction angle \( \phi \) for Ottawa sand is obtained from the relation (Rowe 1969, Kulhawy and Mayne 1990):

\[
\tan \phi_{CS} = \tan 1.12\phi_{p,TX} \cos \phi_c \tag{7}
\]

where \( \phi_{CS} \) is the friction angle from the direct shear test, \( \phi_{p,TX} \) is the peak friction angle from triaxial testing, \( \phi_c \) is the critical-state friction angle. \( \phi_{p,TX} \) can be obtained from Bolton’s relation given by \( \phi_{p,TX} = \phi + 3I_k \). Using Eq. (7), \( \phi \) is obtained as 27 degrees.

Granular backfill (\( c=0 \)) with unit weight \( \gamma = 18 \) kN/m\(^3\) is considered in the study. For the conventional method, a constant friction angle of \( \phi = 38 \) degrees is used in the calculation of active earth thrust, whereas to account for stress dependency on the friction angle, \( \phi \) given by Eq. (5) is used.

**RESULTS AND DISCUSSION**

Retaining wall of height \( H \) is divided into horizontal slices of thickness equal to 0.1 \( H \) and the horizontal thrust on the wall is calculated by using Eq. (4). \( P \) is normalized with \( \gamma H^2 \) and the variation of normalized \( P \) with inclination of slip plane \( \theta \) for various wall heights (5-to-40m) is plotted (Fig. 3). The critical slip plane for a given wall height is then located corresponding to the maximum value of \( P \).

The critical plane is located by plotting the variation of \( P^{*} = \) for various wall heights of 5 to 40 m (Fig. 3). The critical slip plane becomes flatter as the wall height increases. Fig. 4 shows the variation of critical slip plane \( \theta_c \) with the wall height. \( \theta_c \) decreases from 70.2\(^o\) to 65.6\(^o\) as the wall height increases from 5 m to 40 m. Coulomb’s theory gives a value of 64\(^o\) \( (= 45 + \theta/2) \) when a constant friction angle value of 38\(^o\) is used.

Fig. 5 shows the variation of mobilized friction angle with depth. Mobilized friction angle varies from 68\(^o\) to 42\(^o\) for wall height equal to 20 m. High friction angles are mobilized at the top of the wall due to very low effective confining stresses leading to high dilatancy. It may be noted that the locked-in compaction stresses developed during compaction of backfill are not accounted for in the present analysis. Figs. 6a to 6c show the variation of coefficient of lateral earth pressure with depth for wall heights \( H = 5 \) m, 20 m and 40 m, respectively. Fig. 7 show the variation of normalized \( P^{*} = (P/\gamma H^2) \) in active condition for wall heights from 5 to 40 m. Lateral thrust considering the effect of
stress on the mobilized friction angle is found to be lower than that obtained by considering a constant friction angle. As the wall height increases, $P^\ast$ calculated by considering the effect of stress on the mobilized friction angle approaches that of the $P^\ast$ value obtained from Coulomb's theory. The difference in $P^\ast$ calculated using the proposed method decreases from 45% to 9% of the conventional method as the wall height increases from 5 m to 40 m.
CONCLUSIONS

When the effect of confining stress was considered on the mobilized friction angle, the inclination of critical slip plane with the horizontal decreases from 70.2° to 65.6° as the wall height increases from 5 m to 40 m. The lateral thrust estimated with the consideration of stress dependent angle of shearing resistance was lower than that obtained from Coulomb’s theory by 45%-to-9% as the wall height increases from 5 m to 40 m.

REFERENCES