CONSOLIDATION OF SOFT GROUND WITH DRAINS WITH NO (PVDs) AND FINITE STIFFNESS (GRANULAR PILES)

M.R. Madhav, Professor Emeritus, JNTUH CE and Visiting Professor, IIT, Hyderabad, madhavmr@gmail.com
P. Ayub Khan, Associate Professor, ACE Engineering College, Hyderabad, akp1468@gmail.com

ABSTRACT: PVDs and granular piles are the most popular techniques for improving soft soil deposits. While the former improves the ground by accelerating consolidation the latter improves the ground by not only accelerating consolidation but by reinforcing the soft ground as well. This paper presents review of the different analyses especially of non-linear theory of radial consolidation due to PVDs in thick clay deposits and the consolidation of granular pile-reinforced soils considering the stiffness of the granular pile.

INTRODUCTION
India has a long coast line of nearly 5,000 km length. Soft, weak, highly compressible soft soils are prevalent along this stretch. Considerable infrastructure development is taking place along the coast for obvious reasons. Construction of facilities on these deposits is very challenging because of their low strength, high compressibility, etc. Several alternatives, e.g. preloading without or with PVDs, granular piles, heavy tamping, etc., are available to engineer the ground. Amongst these, preloading with PVDs and reinforcing with granular piles/stone columns are the most preferred alternatives.

Prefabricated Vertical Drains (PVDs)
These are band or strip shaped plastic drains about 100 mm wide and 4 mm thick usually installed in square or triangular arrays (Fig. 1a). Consequently, the zone of influence of each drain is either square or hexagonal area. The strip drain and the zone of influence of each drain are replaced by a unit cell of equivalent circular shapes. The flow pattern around the drain is considered to be axi-symmetric (Fig. 1b). The equivalent diameter of the drain, \( d_w = \frac{2(a+b)}{\pi} \), where ‘a’ and ‘b’ are the width and thickness of the PVD respectively and the equivalent diameter of the zone of influence, \( d_e = 1.13S \) & 1.05S for square and triangular patterns respectively, where S is the spacing of drains.

Barron [1] presented analytical solutions for the radial consolidation due to sand drains for both free and equal strain conditions. This classical theory is based on the assumptions of small strains, linear void ratio-effective stress relationship and constant coefficients of volume compressibility, \( m_v \), and horizontal permeability \( k_h \). Hansbo [2] presented a simple solution for radial consolidation with band shaped vertical drains. Lekha et al. [3] presented a non-linear theory of consolidation with sand drains under time dependent loading for equal strain case. Full-scale test conducted by Bergado et al. [4] on soft Bangkok clay with PVDs revealed that the degree of consolidation obtained from pore pressure measurements is lower than the corresponding values obtained from settlement measurements. The various modeling aspects of PVDs are comprehensively discussed by Indraratna et al. [5] along with the evaluation of their effectiveness in practice. Indraratna et al. [6] developed a theory for consolidation with radial flow using \( e - \log \sigma' (C_c \) and \( C_r \) & \( e - \log k_h (C_k) \) relationships and for different loading increment ratios \((\Delta \sigma'/\sigma'_0)\). Indraratna et al. [7] developed a modified consolidation theory for vertical drains incorporating vacuum preloading for both axi-symmetric and plane strain conditions. Two- and three-dimensional multidrain finite-element analyses of a case study of combined vacuum and surcharge preloading with vertical drains is presented by Rujikiatkamjorn et al. [8]. The numerical predictions compared well with observed data. Indraratna et al. [9] developed a new technique to model consolidation by vertical drains beneath a circular loaded area by transforming system of vertical drains into a series of concentric cylindrical drain wall. Walker et al. [10] presented the spectral method for analysis of vertical and radial consolidation in multilayered soil with PVDs by assuming constant soil properties within each layer.

For relatively large applied stress range the void ratio is not proportional to effective stress and the coefficients of compressibility and permeability decrease during consolidation. A non-linear theory of consolidation was developed by Davis & Raymond [11] considering \( e - \log \sigma' \) relationship but it is valid only for vertical flow and thin layer of clay. A theory of non-linear consolidation for radial flow
around a vertical drain is developed by Ayub Khan et al. [12]
based on linear void ratio-log effective stress relationship but
assuming constant coefficient of consolidation for thin clay
layers. This theory is further extended by Ayub Khan et al.
[13] for thick clay deposits as well.
The general equation of non-linear consolidation with radial
flow [13] in terms of a parameter, \( w \), is
\[
\frac{\partial w}{\partial t} = c_f \left( \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right)
\]
with \( w = \log_{10} \frac{\sigma'_{f,j}}{\sigma'_{f}} \) or \( w = \log_{10} \frac{\sigma'_{f,j} - u}{\sigma'_{f}} \), where \( \sigma' \) and \( u \) are
the effective vertical stress and the excess pore pressures
respectively at time, \( t \) and radial distance, \( r \), \( c_f \) - coefficient of consolidation for radial flow and \( \sigma'_{f,j} \) the final effective
vertical stress. The parameter \( \sigma' \) varies with the depth in
thick deposits of clay as the initial effective in-situ stress, \( \sigma'_{o,j} \)
and the final effective stress, \( \sigma'_{o,j} = (\sigma'_{o,j} + q_o) \) vary with depth
due to overburden stresses, and \( q_o \) is the applied load
intensity. The thick clay layer of thickness, \( H \) is divided into
\( m \) layers of thickness, \( \Delta H = H/m \) (Fig. 2). The equation
governing the consolidation process of each layer is
\[
\frac{\partial w_j}{\partial t} = c_r \left( \frac{\partial^2 w_j}{\partial r^2} + \frac{1}{r} \frac{\partial w_j}{\partial r} \right)
\]
where the subscript \( j \) refers to the layer number. Eqs. (1 and
2) are of the same form as that given by Barron [1]. Even
though the initial and final effective stresses are different in
each layer, but the flow in each layer is assumed to be purely
radial and independent of the flows in the adjacent layers.
Degree of Settlement of the layer
\[
U_{s,j} = \frac{\sum_{j=1}^{m} U_{s,j}(\Delta H)}{H} \tag{4}
\]
Normalized average excess pore pressure for the layer \( j \) is:
\[
U_{\text{avg},j} = \frac{r_c}{r_w} \int_{u_j}^{r_c} 2 \pi r \, dr \tag{5}
\]
Degree of dissipation of average excess pore pressure at the
layer, \( j \) is:
\[
U_{P,j} = (1 - U_{\text{avg},j}) \tag{6}
\]
Average degree of dissipation of excess pore pressure for the
entire thickness is:
\[
U_p = \frac{\sum_{j=1}^{m} U_{P,j}(\Delta H)}{H} \tag{7}
\]
Initial and Boundary Conditions
For \( t = 0 \) and \( r_w \leq r < r_e \); \( u_j(r,0) = (\sigma'_{f,j} - \sigma'_{o,j}) \)
or \( w_j(r,0) = w_{o,j}(r,0) = \log_{10} \frac{\sigma'_{o,j}}{\sigma'_{f,j}} \) \( \tag{8} \)
where \( \sigma'_{f,j} = \left(\sigma'_{o,j} + \Delta \sigma'\right) \) and \( \Delta \sigma' = q_o \)
For \( t > 0 \) and \( r = r_w \); \( u_j(r=r_w, t) = 0 \) or \( w_j(r=r_w, t) = 0 \)
\( \tag{9} \)
For \( t > 0 \) and \( r = r_e \); \( \frac{\partial w_j}{\partial r} \bigg|_{r=r_e} = 0 \) or \( \frac{\partial w_j}{\partial r} \bigg|_{r=r_e} = 0 \)
\( \tag{10} \)
\( j=1, 2, 3 \ldots m \)
The radius of influence zone, \( r_e = 0.5d_e \) and radius of
equivalent drain, \( r_w = 0.5d_w \). Eq. (2) is re-written in non-
dimensional form as
\[
\frac{\partial w_j}{\partial T} = \frac{\partial^2 w_j}{\partial R^2} + \frac{1}{R} \frac{\partial w_j}{\partial R} \tag{11}
\]
where \( R = r/d_e \) and time factor, \( T_b = c_e t/d_e^2 \).
depth and radial distance are studied. As non-linear consolidation is mainly influenced by the stress ratio, $\sigma'_f/\sigma'_o$, the variation of $\sigma'_f/\sigma'_o$ with depth is shown in Fig. 3 for different values of normalized applied load intensity, $q^*_o=q_o/(\gamma'H)$. The decrease of $\sigma'_f/\sigma'_o$ with depth is very sharp as the initial effective stress is very small near the top. The decrease of $\sigma'_f/\sigma'_o$ with depth is significant for depths in the range 0.1H to 0.4H and relatively very small for $z>0.4H$ for all $q^*_o$. Hence, the effect of non-linearity in the void ratio - effective stress on consolidation in a thick clay layer can be pronounced at shallow depths compared to that at greater depths. Increases in $q^*_o$ can be either due to increase of load intensity, $q_o$ for a given thickness, $H$, or due to decrease of thickness of clay deposit for a given load intensity. In either case, only the non-dimensional stress ratio, $\sigma'_f/\sigma'_o$, influences the rate of consolidation and not the thickness of the deposit or the magnitude of loading individually.

The degree of settlement, $U_s$, is determined for different layers along the depth for various $q^*_o$ values for different values of $n$ ranging from 5 to 40 and shown in Fig.4. The degree of settlement is identical at all the layers in given thick clay for all values of $n$ and $q^*_o$. The degree of settlement obtained from the present non-linear theory is identical to that from the linear theory [1] for free strain case. The degree of settlement, $U_s$ is independent of $\sigma'_f/\sigma'_o$ and varies only with $n$ as is the case for vertical flow [11]. The degree of settlement decreases with increase of $n$ since it takes relatively long time for dissipation of pore pressure for larger radial distances. $U_s$ decreases from 58% to 24.8% for $n$ increasing from 5 to 40 at a time factor of 0.10.

The degree of dissipation of average excess pore pressure for the entire thickness, $U_p$ is presented in Fig. 5 for $n=15$ along with the degree of settlement, $U_s$. While the degree of settlement is independent of $q^*_o$, the degree of dissipation of pore pressure, $U_p$ is sensitive to $q^*_o$ values. Thus for a given thickness of clay deposit, the increase of load intensity, $q_o$ results in decrease of degree of dissipation of average excess pore pressure or for a given load intensity the decrease of thickness of clay deposit results in decrease of degree of dissipation of average excess pore pressure.

The normalized average excess pore pressure, $U^*_{avg}(z)=(u_{avg}(z)/u_o).100$ at different depths determined and its variation with time shown in Fig. 6 for $n=15$ and $q^*_o=1$ along with the degree of settlement. The excess pore pressure is relatively high at shallow depths where the $\sigma'_f/\sigma'_o$ ratio is relatively high compared to the values at greater depths. The importance of the above finding is that when large load is applied on soft ground, the possibility of shallow seated rotational bearing failure is to be examined in view of the large residual pore pressures at these depths over longer periods of time.
The normalized average excess pore pressures, $U_{avg}(z)$ at different depths of the thick clay are presented in Fig. 7 for $n=15$ at time factor, $T_H=0.20$ along with the results of linear theory. The remarkable phenomenon observed is that average pore pressure values from the non-linear radial consolidation theory vary with depth in contrast to the depth-independent $U_{avg}$ values of linear theory. The differences between the pore pressures of non-linear and linear theories are relatively large at shallow depths due to large values of $\sigma'_f / \sigma'_o$ compared to those at greater depths. This difference increases with increase of $q*o$. Moreover, at shallow depths the variation of pore pressure with depth is relatively very large compared to that at greater depths due to sharp variation of $\sigma'_f / \sigma'_o$ at shallow depths. While the excess pore pressures in the linear theory are independent of, $q*o$, the pore pressures according to non-linear theory are dependent on $q*o$ as the variation of $q*o$ influences the ratio $\sigma'_f / \sigma'_o$.

The residual average excess pore pressures thus are underestimated in the conventional linear theory. In view of the above, instead of applying the entire preload instantaneously on the soft ground, it may be applied in increments with proper time lag to allow quicker dissipation of pore pressures and gain of shear strength.

Fig. 8 shows the pore pressure variation with radial distance at different depths for $q*o=1$, $n=15$ and $T_H=0.20$. The excess pore pressures are relatively large at shallow depths wherein the stress ratio is extremely large but decrease with depth since the ratio of final to initial stress decreases with increase of depth. The pore pressure variation with radial distance, $r$, is relatively more significant in the upper half of the deposit compared to that in the lower half.

**Fig. 6** Variation of $U_s$ & $U_{avg}(z)$ with $T_H$—Effect of Depth

**Fig. 7** Variation of $U_{avg}(z)$ with Depth—Effect of Load Intensity

**Fig. 8** Variation of Pore Pressures with Radial Distance—Effect of Depth

**GRANULAR PILES (GP)**

Granular piles (GP)/stone columns increase bearing capacity of foundations laid on the ground, reduce settlement and accelerate the rate of consolidation of soft soils. The increase of bearing capacity and reduction of settlement are attributed to the higher stiffness of GP compared to that of the surrounding soil. The acceleration of consolidation is credited to higher or larger permeability of stone columns and shorter drainage path. In addition, GPs relieve excess pore pressures by transferring loads from soft soil to GP due to the latter’s higher stiffness. Typical drained deformation modular ratio of GP to soft clay ranges from 10 to 20, Han and Ye [14]. The increase of modular ratio of GP to soil accelerates the consolidation rate under a rigid raft but not under a flexible one [15]. Though the PVDs and GPs involve radial flow, basic differences between them are (i) GPs have higher deformation modulus than the surrounding soil while the PVDs have no stiffness and cannot sustain any loads.
The classical solutions for radial consolidation developed by Barron [1] and Hansbo [2] do not consider the effect of stiffness of the drains as they do not have any. (ii). Granular piles have smaller diameter ratio (= unit cell/GP diameters) compared to that for PVDs. Typical diameter ratios for stone columns range from 1.5 to 5 while the range varies from 10 to 30 in practice in the case of PVDs. Han and Ye [16] and Xie et al. [17] observed that the effectiveness of GPs as drains to accelerate consolidation is reduced due to contamination of stones with fine grained soil, the disturbance of surrounding soil and the intrusion of stones into the surrounding soil.

GP reinforced soil is analyzed by treating it as an axisymmetric problem (Figs. 1 & 9), similar to that of consolidation due to radial flow towards sand drain in fine grained soil. The classical solution of Barron [1] for radial flow and Terzaghi for 1D consolidation except for the modified coefficients of consolidations used herein. The overall rate of consolidation due to the combined effect of radial and vertical consolidation is expressed as

\[ U_{rv} = 1 - (1-U_r)(1-U_v) \] (15)

The stress concentration ratio, \( n_s \), is defined as [14]

\[ n_s = \frac{\sigma_{cs}}{\sigma_{ss}} = \frac{m_{v,s}}{m_{v,c}} = \frac{E_c}{E_s} \] (16)

where \( \sigma_{cs} \) and \( \sigma_{ss} \) are the vertical stresses within the GP and the surrounding soil respectively, \( E_c \) and \( E_s \) the deformation moduli of GP and the surrounding soil respectively, \( \zeta \) the Poisson’s ratio factor which depends upon the Poisson’s ratios of the GP and the surrounding soil. The variation of stress concentration ratio with the modular ratio (=\( E_c/E_s \)) is depicted in Fig. 10 along with the results of experimental study by Barksdale and Bachus [18]. The stress concentration ratio increases with the increase in the modular ratio. However, many field studies have shown that the stress concentration ratios for GP-reinforced foundations to be in the range of 2 to 6, Mitchell [19].

The increase in effective stress in the soft soil over a certain time period is

\[ \Delta \bar{\sigma}' = u_o U_r \frac{1 - a_s}{1 + a_s (n_s - 1)} \] (17)

where, \( u_o \) is the initial excess pore water pressure, \( U_t \) the degree of consolidation for radial flow. The variations of average total stress on the soil and the GP with time are depicted in Fig. 11 for \( n_s \) of 5 and diameter ratio, N of 5.
Since it is assumed that the entire load applied is initially taken by water in the surrounding soil, the stress on the column (GP) increases from zero and the average stress on the soil is slightly more than the applied pressure, $p$, over the influence area. The stress on GP increases while the stress on the soil decreases with time and consolidation of soft soil. This phenomenon of stress transfer from the soil to GP is referred to "stress concentration/transfer". The commencement of consolidation initiates the process of vertical stress transfer from the soft soil onto the GP. Thus the stress concentration ratio is zero at the beginning and increases with time and approaches the steady stress concentration ratio, $n_s$, at the end of consolidation.

Fig. 12 shows that excess pore pressure in the soft ground reduces because of vertical stress transfer. The magnitude of this reduction can be about 40% of the total applied stress for the parameters considered. The phenomenon of vertical stress transfer from soft soil to the drain doesn’t exist in the case of PVDs as the latter have no stiffness. Higher stress concentrations induce rapid dissipation of pore pressure

$$U = 1 - e^{-\frac{8T_{rm}}{F_m}}$$

where the modified time factor for radial flow, $T_{rm} = \frac{c_{rm}t}{d_c^2}$, $c_{rm}$ the modified coefficient of consolidation for radial flow, the same as $c_r$ in Eq. (13).

$$F_m' = \frac{N^2}{N^2 - 1} \left[ \ln \frac{N}{S} + \frac{k_r}{k_s} \ln S - \frac{3}{4} \right] + \frac{S^2}{N^2 - 1} \left[ 1 - \frac{k_r}{k_s} \right] \frac{1 - S^2}{4N^2}$$

$$+ \frac{k_r}{k_s} \frac{1}{N^2 - 1} \left[ 1 - \frac{1}{4N^2} \right] + \frac{32}{\pi^2} \left( \frac{k_r}{k_c} \right) \left( \frac{H}{d_c} \right)^2$$

where $S$ - the ratio of diameter of the smeared zone ($d_s$) to the diameter of GPs ($d_c$), $k_r$ and $k_s$ – the coefficients of radial permeability of the undisturbed soil and smeared soil respectively, $H$ the longest drainage distance in the vertical direction. Eq. (19) is similar in form to that by Hansbo [2]. However the present equation takes into account the characteristics of GP-reinforced soil. The effect of some of the important parameters incorporated in Eq. (19) is discussed below.

The influence of the diameter ratio on the rate of consolidation is presented in Fig. 13 for typical diameter ratios of 1.5 to 5. The rate of consolidation is accelerated with the decrease in the diameter ratio due reduction in the drainage path. The diameter ratio can be decreased either by decreasing the spacing between the GPs or/and by enlarging the diameter of the GPs.

The effect of the stress concentration ratio on the rate of consolidation of GP treated ground is depicted in Fig. 14. Consolidation is accelerated with increase in stress concentration ratio.
Consolidation of soft ground with drains with no (PVDs) and finite stiffness (granular piles)

The theory proposed by Han and Ye [16] considers the excess pore pressures developed even inside GPs unlike in the classical theories of Barron [1] and Hansbo [2]. The rate of dissipation of excess pore pressures within the GP depends on the length of the column. This aspect is embedded in the function $F_m$. As expected, the increase in thickness of soft soil retards the rate of consolidation due to an increase of the drainage path for water draining out after it enters the GP. Fig. 15 depicts the reduction of rate of consolidation due to increase in the thickness of soft soil for a given set of parameters involved.

During consolidation the average vertical effective stress in the GP increases while the average vertical total stress in the soil decreases with time due to stress transfer from soil to GP. This change of stress is illustrated in Fig. 16. In the case of PVD treated ground, the entire stress applied is borne by the soft soil alone and the total stress remains constant during the consolidation process.

The comparison of different methods to evaluate the rate of consolidation of GP-reinforced soil is shown in Fig. 17. The
solutions used herein, Han and Ye [14] and Barron [1] ignore the smear and well resistance effects while the other two solutions, Han and Ye [16] and Hansbo [2] consider the smear and well resistance effects. Han and Ye [14] solution overpredicts the rate of consolidation compared to that by Barron due to the reason that the former theory considers the stiffness of GP or the stress concentration ratio which accelerates the rate of consolidation. The same reason accounts for the faster the rate of consolidation predicted by Han and Ye [16] compared to that by Hansbo [2] even though both the theories consider smear and well resistance effects.

CONCLUSIONS
Consolidation of soft ground treated by PVDs and granular piles are compared and contrasted. The former are soft and cannot carry any of the applied stress while the latter accelerate consolidation further by transfer of load from soft ground to stiff granular piles during the consolidation phase. Thus consolidation by GPs is extremely fast compared to that by PVDs. A simple non-linear theory of radial consolidation developed for thick deposit of clay treated with PVDs considering linear void ratio-log effective stress relationship predicts that while the degree of settlement is independent of the final to initial stress ratio, the degree of dissipation of pore pressure is very much dependent on the stress ratio. The residual excess pore pressures are under-estimated in the conventional linear theory. The proposed nonlinear theory substantiates the actual in-situ slower rate of degree of dissipation of excess pore pressures compared to that of the degree of settlement. The non-linear consolidation effect is pronounced at shallow depths compared to the effect at greater depths. The excess pore pressures due to radial drainage vary not only with time and radial distance but also with depth in contrast to depth-independent pore pressures from the conventional theory for radial flow. The significance of the proposed theory is that it can explain failure of high embankments constructed rapidly on thick deposits of fine grained soils.

The rate of consolidation due to GPs can reasonably be predicted considering the modular ratio of GPs to that of the soil, the smear and well resistance effects. The smear and well resistance effect is significant in the case of GPs. The rate of consolidation is accelerated because of increasing stress concentration ratio.

REFERENCES