Optimal Design of Composite Channels Under Slope Stability Criterion

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ABSTRACT

Canals are major water conveyance systems used to deliver water to irrigation fields. Due to wide variability of local soil conditions, the stability of designed channel section may not be ensured while implementing the optimal canal sections in the real field. Hence, incorporation of slope stability condition is very helpful to provide stable irrigation channels. This paper presents Particle Swarm Optimization (PSO) based solutions for optimal design of composite trapezoidal channels after incorporating the slope stability criterion. The method is applied for different soil conditions to show its practical applicability. The solutions are compared with those obtained by a hybrid optimization procedure involving genetic algorithm (GA) and sequential quadratic programming (SQP). The solutions of PSO method are found to be competitive to the existing solutions. This study suggests that the proposed approach for stability constrained channel design is effective and can be used for field problems.

1. INTRODUCTION

Artificial channels are used as the major conveyance systems to deliver water to irrigation fields. As the channels run for long distances to serve its purposes, irrigation projects become costlier undertakings. In order to achieve the cost effectiveness in construction of the channels, it is common practice to use different lining materials along the channel perimeter. Such channels with distinctly different lining materials are called as ‘composite channels’ (Chow, 1959). The channel roughness is a major factor which decides the conveying capacity of canals. For composite channels the surface roughness is accounted by introducing an equivalent roughness coefficient in any of the uniform flow formula for discharge computation. Das (2000) determined the optimal trapezoidal channel cross-section with composite roughness using classical optimization technique involving Lagrange Multipliers (LM).

Later on many researchers attempted the optimal design of composite channels using meta-heuristic optimization techniques (Jain et al., 2004, Nourani et al, 2009, Adarsh & Janga Reddy, 2010). To implement the designed channel geometries in the real field, the stability of local soil conditions also needs to be considered as it may lead to failure of channels. This issue was addressed earlier by Bhattacharya & Satish (2007). They used a hybrid optimization methodology involving Genetic Algorithm (GA) and the classical Sequential Quadratic Programming (SQP) to solve the stability constrained channel design problem. In this paper, a recent meta-heuristic optimization method namely Particle Swarm Optimization (PSO) is used for the determination of optimal dimensions of composite channels considering slope stability as an additional constraint in the channel design model.

2. PARTICLE SWARM OPTIMIZATION

Particle swarm optimization is a population based random search optimization technique, inspired by social behavior of bird flocking. It was proposed by Kennedy & Eberhart (Kennedy & Eberhart, 1995). The social sharing of information among individuals is the fundamental hypothesis of PSO. Like other evolutionary computation techniques, PSO algorithm is initialized with random solutions. Particles fly through search space with the velocities which are dynamically adjusted according to their historical behavior.

The dimension of the problem will be the number of decision variables involved. In a ‘D’ dimensional search space, the \( i^{th} \) particle can be represented as a ‘D’ dimensional vector \( X_i = (x_{i1}, x_{i2}, ..., x_{iD})^T \). The velocity (position change) of this particle is designated as \( V_i = (v_{i1}, v_{i2}, ..., v_{iD})^T \). The best previously visited position of the \( i^{th} \) particle is given by \( P_i = (p_{i1}, p_{i2}, ..., p_{iD})^T \). Defining ‘g’ as the index of the best
particle in the swarm and superscripts denoting the iteration number, the population dynamics can be described using the following equations:

$$v_{id}^{n+1} = \omega v_{id}^{n} + a_1(p_{id}^{n} - x_{id}^{n}) + a_2(r_{1d}^{n} - x_{id}^{n})$$  \hspace{1cm} (1)

$$x_{id}^{n+1} = x_{id}^{n} + v_{id}^{n+1}$$  \hspace{1cm} (2)

where $d = 1, 2, ..., D$, the index for decision variables, $i = 1, 2, 3...N$, the index for swarm population, $N$ is the size of the swarm; $a_1$ and $a_2$ are called acceleration coefficients; $r_{1d}$ and $r_{2d}$ are random numbers between zero and 1; $\omega$ is the inertia weight; $\chi$ is the constriction coefficient.

The control parameters of the algorithm play a vital role in dering good quality global optimal solutions. A more detailed description of the algorithm can be found in Nagesh Kumar & Janga Reddy (2007).

3. OPTIMAL DESIGN MODEL FOR COMPOSITE CHANNELS

The non-linear optimization model for cost effective design of a trapezoidal channel considers the minimization of construction cost. In this study, excavation cost and lining cost are considered as the two construction cost elements. The roughness parameters of the sides of the channel and the bottom are assumed to be different. Manning’s discharge equation appears as an equality constraint in his model. Due to the non-linearity and non-convexity of the model a meta-heuristic optimization technique is chosen as a suitable method to get the optimal solution. For solving in a meta-heuristic optimization framework, the optimization model can be represented as follows:

Minimize:

$$C(b, y, z_1, z_2) = c_e A_1 + c_1 P_1 + c_2 P_2 + c_3 P_3$$  \hspace{1cm} (3)

Subject to:

$$g_1(b, y, z_1, z_2) = \varepsilon - \frac{Q n_y}{\sqrt{S}} - \frac{A_w^{5/3}}{P_w^{2/3}} \geq 0$$  \hspace{1cm} (4)

where, $C =$ total construction cost of channel per m length; $c_e =$ the excavation cost per unit cross sectional area for unit length of channel; $c_1$, $c_2$, and $c_3$ are lining cost per unit length of the perimeter segment 1, 2 and 3 respectively for unit length of channel. $P_1$, $P_2$ and $P_3$ are the perimeters of the two side slopes and the bottom of the channel respectively; $b =$ base width of the channel, in m; $y =$ flow depth, in m; $z_1 = l$ and $z_2 = l$ are the side slopes of the channel $Q =$ design discharge of the channel, in m$^3$/sec; $A =$ flow area, in m$^2$; $P_w =$ total wetted perimeter, in m; $S =$ longitudinal slope of the channel; $A_1 =$ total channel cross sectional area, in m$^2$; $n_e =$ the equivalent roughness coefficient of the composite channel calculated by Horton’s method (1933) and $\varepsilon$ is a very small positive real number.

4. SLOPE STABILITY CONSTRAINED CHANNEL DESIGN

The practical implementation of the designed canal sections in real field is a challenging task as the issue of stability of the side slope must be ensured. The stability of side slope of canal is highly dependent on the local soil conditions. In most cases, the practitioners select the side slope considering the angle of repose of the soil. This may not be a proper decision as there can be a wide variability in soil properties at different site conditions along the channel. The channel designed may be optimal and stable for a particular set of soil parameters, but it may not be stable for some other local soil conditions. This issue can be addressed by incorporating a rigorous slope stability constraint in the optimal design of canals. For practical purposes it is preferable to adopt equal side slopes $(z_1 = z_2 = z)$. Thus the stability constrained channel design model can be formulated by introducing the following additional constraint into the above model consisting of Eq. (3) and (4), along with $z_1 = z_2 = z$.

$$g_2(b, y, z_1, z_2, \phi, \gamma) = F \geq F_m$$  \hspace{1cm} (5)

where, $c =$ cohesion of soil, in kN/m$^2$; $\phi =$ angle of internal friction of soil; $\gamma =$ bulk density of soil and $F =$ factor of safety; $F_m =$ minimum required factor of safety.

From the practical point of view, a factor of safety of 1.5 is considered to be indication of a stable slope. The additionally introduced constraint (Eq. 5) in the optimization model for channel design ensures a minimum factor of safety of 1.5 for the optimal channel section. The resulting model is solved using PSO by considering the slope stability constraint as a separate sub-optimization formulation.

Features of Slope Stability Model

In this study, the slope stability sub-optimization model is developed based on the following assumptions:

(1) soil is homogeneous and isotropic (2) failure surface is circular (3) base of the canal is rigid and the failure arc passes through the toe of the slope (4) as the canal is lined, the seepage is neglected and the soil surrounding the channel section is considered to be unsaturated, i.e., bulk unit weight is used in the calculation of factor of safety and effect of pore pressure is neglected. Figure 1 shows a homogeneous channel slope with a trial circular slip surface which passes through the toe. The figure also describes different decision variables involved in the channel optimization model.

The PSO based sub optimization model takes the soil parameters like cohesion, angle of internal friction and bulk density of soil. The different forces in the expression for
factor of safety by Felleneus (1936) are expressed in terms of the centre of co-ordinate of assumed slip circle, which appear as the decision variables of the sub-optimization formulation. The bounds on the decision variables are the constraints of sub-optimization model.

Minimize: \[ F(X, Y) \]
Subject to \[ X_L < X < X_U \]  \hspace{1cm} (6)
\[ Y_L < Y < Y_U \]  \hspace{1cm} (7)

where, \( X_L \) and \( X_U \) are lower and upper limit on \( X \) co-ordinate of centre of slip circle; \( Y_L \) and \( Y_U \) are lower and upper limit on \( Y \) co-ordinate of centre of slip circle.

**Optimal Solutions for Slope Stability Constrained Channel Design Model**

The solution procedure of slope stability constrained channel design model is illustrated with the help of a flow chart in Figure 2. Only the trial solutions (particles) which satisfy the Manning’s equality constraint are considered for the estimation of factor of safety. The value of flow depth \( y \) (along with a fixed freeboard of 0.5 m) and side slope parameter \( z \) from the ‘swarm’ are passed to the slope stability model, which defines the geometry of the slope. For a specified soil condition (i.e., for a given value of \( c, \delta \) and \( \gamma \), the factor of safety of the defined slope geometry is estimated using PSO algorithm. The result of this sub-optimization model (i.e., minimum factor of safety of the defined geometry) is imported back to the main program and it is checked for the minimum required factor of safety value of 1.5. On satisfying the stability constraint, the channel design model is solved for minimization of cost using PSO algorithm. In between any of the constraint is violated by some particle, a suitable penalty is given for the fitness and such particles are eliminated from the population.

The slope stability constrained model is solved for nine different combinations of soil parameters. The data is taken from Bhattacharjya & Satish (2007); and the results are compared with the solutions obtained by SQP-GA hybrid procedure suggested by the same authors. The soil parameters constitute a variety of field conditions. The first three cases are for \( Q = 100 \text{ m}^3/\text{sec} \); next three cases for \( Q = 200 \text{ m}^3/\text{sec} \) and the remaining cases are for \( Q = 500 \text{ m}^3/\text{sec} \). The optimal PSO control parameters used are: swarm size \( (N) = 100 \); inertia weight \( (\omega) \) = linearly varying from 1.2 to 0.4; acceleration coefficients \( (a_1 \text{ and } a_2) \): 1 and 0.5; constriction coefficient \( (\gamma) \) of 0.9. The results are given in Table 1.

**Discussion**

The designed geometry gets wider acceptability by ensuring the stability of channels. The solution of coupled slope stability and channel optimization model for the wide range of soil parameters shows its potential for field applicability. From the results obtained, it can be inferred that the total construction cost increases for weak soil parameters which establishes the conclusions derived by Bhattacharjya & Satish (2007). The comparison of the results obtained from PSO based solution methodology with the existing method of hybrid model of SQP and GA by Bhattacharjya & Satish (2007) clearly reveals that PSO based solutions shows equally good performance with the hybrid model and sometimes better in terms of quality. Thus PSO technique has the capability in handling the complexities of stability constrained composite channel design problem without any compromise on quality. It encourages the use of PSO for complex channel design problems.
5. CONCLUSIONS

This study investigated the usefulness of PSO for optimal design of composite trapezoidal channels considering slope stability constraint. The complexity of channel design model increases on incorporating the factor of safety constraint to ensure the stability of side slopes. The channel design model is coupled with slope stability sub-optimization model and the resulting model is solved using PSO. The proposed approach shows its effectiveness by giving good quality solutions to wide range of soil conditions, and demonstrates its potential for field applicability. Thus the method presented in this paper can be used effectively for designing stable composite channels.

REFERENCES


| Table 1: Optimal Solutions for Stability Constrained Channel Design Model |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| S.No | c (kPa) | \( \Phi \) (degrees) | \( \gamma \) (kN/m^3) | Centre of Coordinate | Factor of Safety | b(m) | y(m) | \( z_1, z_2 \) | Cost (Rs 1000/m) |
| (a) \( Q = 100 \text{ m}^3/\text{sec} \) | | | | | | | | |
| 1 | 20 | 20 | 18 | (-3.77,7.19) | 1.534 | 5.816 | 4.056 | 0.256 | 22.96 | 22.95 | PSO |
| 2 | 10 | 20 | 18 | (-0.06,4.69) | 1.499 | 6.797 | 3.005 | 0.831 | 24.51 | 24.99 | GA-SQP* |
| 3 | 5 | 20 | 18 | (2.14,6.49) | 1.499 | 4.025 | 3.075 | 1.979 | 28.59 | 28.73 |
| (b) \( Q = 200 \text{ m}^3/\text{sec} \) | | | | | | | | |
| 1 | 10 | 28 | 18 | (-0.32,6.39) | 1.499 | 6.918 | 4.349 | 0.910 | 38.00 | 38.21 | PSO |
| 2 | 5 | 28 | 18 | (0.86,7.14) | 1.499 | 7.665 | 3.754 | 1.487 | 41.45 | 41.64 | GA-SQP* |
| 3 | 20 | 10 | 18 | (-0.03,6.53) | 1.499 | 7.172 | 4.527 | 0.692 | 37.03 | 37.37 |
| (c) \( Q = 500 \text{ m}^3/\text{sec} \) | | | | | | | | |
| 1 | 10 | 28 | 18 | (0.66,8.67) | 1.499 | 9.421 | 5.848 | 1.211 | 72.47 | 72.73 | PSO |
| 2 | 5 | 28 | 18 | (1.77,10.50) | 1.499 | 8.175 | 5.610 | 1.783 | 77.94 | 77.94 | GA-SQP* |
| 3 | 10 | 5 | 20 | (14.35,14.54) | 1.499 | 8.349 | 4.055 | 6.003 | 112.51 | 113.4 |

Note: *Present Study; *Bhattacharjya and Satish (2007)