Stability of Tall Structures Founded on Soft Inelastic Ground

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ABSTRACT

The governing equations for stability of tall structures are derived and expressions for critical height obtained considering inelastic response of the ground, i.e., its modulus of deformation in unloading is different from the corresponding value in compression and Winkler model to simulate the subsoil response. The effect of the ratio of soil modulus in unloading to that in compression on moment – rotation relationship is quantified. Critical height of the structure is expressed in terms of a non-dimensional stability number. A parametric study of the factors influencing the stability number is presented.

1. INTRODUCTION

Leaning instability is the most probable cause of tilting and toppling of tall structures like towers, smokestacks, etc. If a critical height, which is a function of the stiffness of the subsoil and the geometry of the foundation, is attained, the structure reaches a state of neutral equilibrium and eventually may fall even under a small disturbance. It should be noted that leaning instability is not due to lack of strength of ground but due to insufficient stiffness. Analysis of leaning instability has been carried out Habib and Puyo (1970); Hambly (1985); Lancellotta (1993) and Desideri and Viggiani (1994) based on elastic response of the ground. In this paper, the stability of initially vertical tall structures is derived and expressions for critical height found out with the subsoil having its modulus of deformation in compression different from the value in unloading.

2. STATEMENT OF THE PROBLEM

A rigid footing subjected to an applied moment, \( M \), is considered (Fig.1(a)). The ground below one part of the footing undergoes compression while the other part undergoes unloading or stress reversal. A modular ratio, \( R \), is defined as \( R = k_c/k_u \), where \( k_c \) and \( k_u \) are moduli of subgrade reactions in compression and unloading respectively.

3. ANALYSIS FOR ROTATION BASED ON WINKLER MODEL AND LINEAR RESPONSE

Rectangular Footing

A rectangular footing of width, \( B \), and length, \( L \), subjected to moment, \( M \) and its deformation response are shown in

Figures 1(a) and (b). The distance to the axis of rotation from the left edge of footing (Fig.1(c)) is \( x_0 \). The footing rotates through an angle \( \theta \) due to the moment, \( M \), applied. The stress on the compression side (Fig.1(c)) is \( k_c \delta \) where \( \delta = \theta x \) is the deformation at distance, \( x \) from the axis of rotation. Similarly, the stress on the footing on the unloading side is \( k_u \delta \) or \( k_u \theta x \). Negative sign indicates unloading. Since a footing subjected to only moment is considered no vertical force acts on the footing. Force equilibrium leads to
\[ 0.5(L - x_0)k_c \theta (L - x_0) = 0.5x_0 \theta x_0 k_u \]  \quad (1)

Solving
\[ X_0 = x_0/L = \left\{ 1 - \sqrt{(R_1)}/(1 - R_1) \right\} \]  \quad (2)

For \( R_1 = 1.0 \), \( X_0 = 0.5 \) as is to be expected. Taking moments of the forces with respect to the axis of rotation
\[ 0.5(L - x_0)k_c \theta \frac{2L - x_0}{3} + 0.5x_0 \theta \frac{2x_0}{3} k_u = M/B \]  \quad (3)

or \[ (1/3)(1 - X_0)\beta + R_cX_0^3 \theta = M/(k_cBL^4) \]  \quad (4)

The rotation is expressed as \( \theta = \frac{M}{k_cBL^4} \)  \quad (5)

where \( I_{boc} = 3/[1 - (1 - X_0)\beta + R_cX_0^3] \)
\[ X_0 = x_0/L \]  \quad (6)

Circular Footing

A circular footing of radius, \( R \), is subjected to moment, \( M \). Its displacements are shown in Figures 2(a) & (b). The corresponding equations for force and moment equilibrium are respectively
\[ \int_0^\infty 2k_c \theta x + D)Cdx + \int_0^\infty 2k_c \theta (D - x)Cdx - \int_\infty^0 2k_c \theta x - D)Cdx = 0 \]  \quad (7)

\[ \int_0^\infty 2k_c \theta x + D)Cdx + \int_0^\infty 2k_c \theta (D - x)Cdx - \int_\infty^0 2k_c \theta x - D)Cdx = M \]  \quad (8)

where \( C = \int (R^2 - x^2)D \), \( D = R - x_0 \). The integration is carried out numerically and the resulting expression is \[ \theta = M^*I_{boc} \]  \quad (9)

where \( M^* = \frac{M}{k_c d^4} \) ; \( d \) - the diameter of the footing, \( I_{boc} \) - a function of \( X_0 = x_0/R \) and \( R_c \).

Figure 3 presents the variation of \( X_0 \), the distance to the axis of rotation for rectangular and circular footings with \( R_c \). The axis of rotation passes through the midpoint of the length of rectangular or diameter of the circular footing, i.e. \( X_0 = 0.5 \) for \( R_c = 1 \) (elastic response). The axis of rotation shifts towards the zone of stress reduction with increasing inelasticity or \( R_c \) values. The rate at which the axis of rotation shifts with increase in \( R_c \) is high for \( R_c < 5 \) for both rectangular and circular footings. The rate of decrease of \( X_0 \) with \( R_c \) decreases gradually for \( R_c > 5 \) though the rate of shift of the axis of rotation is nearly the same for both types of footings for \( R_c < 2 \), the rate of decrease is somewhat higher for rectangular footing for higher \( R_c \). For example, at \( R_c = 20 \), \( X_0 \) shifts by about 32% of its length for rectangular footing, while it is only 25% of the diameter for circular footing.

Figure 4 depicts the variation of \( I_{boc} \) with \( R_c \) for circular and rectangular footings. \( I_{boc} \) decrease with increase in \( R_c \) for both footings. The results indicate smaller rotations for higher \( R_c \). The rate of decrease in \( I_{boc} \) with \( R_c \) reduces with increase in \( R_c \). The values of \( I_{boc} \) are 12 and 20.37 for \( R_c = 1 \), for rectangular and circular footing respectively. These values match exactly with those from Weismann (1972).
types of footings. The rate of decrease in normalized
Unstable Condition. Critical height corresponds to a
condition of neutral equilibrium, which is given by
Rectangular Footing
its value at \( R_k \) is nearly the same for circular and
rectangular footings. The reduction of \( I_\theta \) is about 65% of
its value at \( R_k = 1 \) for circular footing and about 63% for
rectangular footing for \( R_k = 20 \).

4. LEANING INSTABILITY OF TALL
STRUCTURES
The concept has been applied to tall structure, with weight,
\( W \), and centre of gravity at a height, \( h \), above ground level
as shown in Figure 6. If the structure is arbitrarily disturbed
through an angle, \( \Delta \theta \), the lateral displacement of centre of
gravity will cause a driving moment, \( W.h.\sin(\Delta \theta) \), while
the base of the footing will produce a resisting moment
equal to \( k_e \Delta \theta \), where \( k_e \) is the rotational stiffness of the soil.
\( k_e \) is obtained from moment-rotation relationships based
on Winkler model response. The difference between the
two moments corresponds to the net restoring moment \( M_r \),
that is
\[
\kappa_e \Delta \theta - W.\sin(\Delta \theta) = M_r
\]
(10)
For small \( \theta \), \( \sin(\Delta \theta) \approx \Delta \theta \). A positive restoring moment
indicates a stable condition, whereas a negative restoring moment
represents an unstable condition. Hence,
If \( k_e > W.h \) – Stable Condition and if \( k_e < W.h \) –
Unstable Condition. Critical height corresponds to a
condition of neutral equilibrium, which is given by
\[
h_{cr} = k_e/W
\]
(11)
The structure starts to lean when the above equation is
satisfied. If \( h > h_{cr} \), the structure will topple.

**Rectangular Footing**
From the Eq. 5
\[
\Delta \theta = \frac{\Delta M}{k_eBL^3}
\]
(12)
\[
k_e = \frac{\Delta M}{\Delta \theta} = \frac{k_eBL^3}{I_{bew}}
\]
(13)
Substituting \( k_e \) from Eq. 8 and rearranging
\[
\frac{W}{k_e} = \frac{L^2}{I_{bew}} = \frac{q}{k_e} = \frac{L^2}{I_{bew}}
\]
(14)
where \( q = (W/LB) \) is the uniform soil pressure due to
vertical load, \( W \). If \( S \) be the average settlement of the
foundation, \( q/k_e = S \), and the above equation can be written
in the following form
\[
S = \frac{1}{L^2} \frac{1}{I_{bew}}
\]
(15)
For rectangular foundation, the radius of gyration \( r_c = L/\sqrt{12} \)
corresponding to the axis of rotation through centre
of area of footing and perfect elastic condition (\( R_k = 1 \)). Hence,
Eq. 12 becomes
\[
S_{cr} = h_{cr} S/r_c^2 = 12/I_{bew}
\]
(16)
where \( S_{cr} \) is the stability number which is similar to that
given by Hambly (1985).

**Circular Footing**
The moment –rotation relationship is
\[
\Delta \theta = \frac{\Delta M}{k_e BL^3}
\]
(17)
At neutral Equilibrium
\[
k_e = \frac{\Delta M}{\Delta \theta} = \frac{k_e d^4}{I_{bew}} = W h_{cr}
\]
(18)
Rewriting Eq. 18 after substituting for \( k_e \)
\[
\frac{4d^2}{\pi I_{bew}} = \frac{q}{k_e} \Rightarrow \frac{S h_{cr}}{d^2} = \frac{4}{\pi I_{bew}}
\]
For circular footing, the radius of gyration, \( r_c = d/4 \),
corresponding to \( R_k = 1 \). Replacing \( d \) in the Eq. 18 in terms
\( r_c \), one gets
\[
S_{cr} = h_{cr} S/r_c^2 = 64/I_{bew}
\]
(19)
Figure 7 shows the variation of stability number with
\( R_k \) for rectangular and circular footings based on in-elastic
response of the ground. Stability number increases with
\( R_k \), the rate of increase in stability number with \( R_k \) is very
high for \( R_k \) in the range of 1 to 5 for both rectangular and
circular footings. With further increase in \( R_k \), the rate of
increase of \( S_{cr} \) with \( R_k \) reduces gradually. The values of \( S_{cr} \),
are 1, 1.8, 2.4 and 2.8 for \( R_k \) equal to 1, 5, 10 and 20
respectively for rectangular footing. The variation of \( S_{cr} \)
with \( R_k \) for a circular foundation is very similar to that for
rectangular footing. However, the values of \( S_{cr} \) for circular
foundation are slightly larger than those for rectangular
foundation and the difference increases with \( R_k \).

5. CONCLUSIONS
Rotation of a rigid footing due to an applied moment is
lesser, if the modulus in unloading is greater than the one
in compression. Rotation decreases with increase in \( R_k \).
The axis of rotation of the footing is at the center of the
footing for elastic response (\( R_k = 1 \)), but shifts towards the

**Fig. 5: Normalised \( I_\theta \) Verses \( R_k \) for Rectangular and Circular
Footings – Winkler Model**

- Neutral Equilibrium
- Critical height
- Unstable Condition
- Stability of Tall Structures Founded on Soft Inelastic Ground
- Rotation decreases
- Unloading modulus
- Elastic response
- Axis of rotation
- Centroid
- Critical condition
- Ground response
- Elastic condition
lifted edge of the footing for increasing $R_k$. Critical height of the structure for leaning instability increases with increase in modular ratio, $R_k$, with the rate of increase with $R_k$ decreasing gradually.

![Diagram of footing](image)

**Fig. 7:** Effect of $R_k$ on Stability Number for Rectangular and Circular Footings

### REFERENCES


