Movement Rotation Relationships of Foundations Using Hyperbolic Subgrade Response

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ABSTRACT

The paper presents response of rigid foundations to combined vertical load and moment considering inelastic response of ground on which they are founded and based on nonlinear Winkler model. The foundation undergoes a uniform settlement due to vertical load and rigid body rotation due to the moment loading. Part of the foundation gets unloaded due to moment loading. The non-linear (hyperbolic) response of the soil is considered for the zone of soil which is in compression due to applied moment. The modulus of subgrade reaction in unloading is different from the modulus in compression. The effect of modular ratio, i.e., ratio of unloading to loading subgrade moduli and intensity of vertical load on rotation are quantified.

1. INTRODUCTION

Tall structures are very often subjected to moments due to wind or other dynamic forces which can cause the structures to tilt as a whole. All the analyses so far available assume that the ground is elastic, i.e., the modulus of deformation under compression is the same as that for stress removal or unloading. However it is well known that these two moduli are not the same. During compression, the soil undergoes both plastic and elastic deformations, while only the elastic part of the total deformation is recovered during unloading. Consequently, the modulus for unloading is much larger than that for compression. Numerous experimental results from plate and pile load tests have established this type of behavior of soil.

2. REVIEW OF LITERATURE

Most of the relationships for determining rotation of rigid footing due to moment are obtained on the basis of Winkler (one parameter) or the elastic continuum models based on Boussineq’s or Mindlin’s expressions. Weismann (1972) derives expressions using Winkler’s model, for tilt of rigid foundations of rectangular and circular shapes. Rotation due to moment loading on smooth, rigid circular footing is obtained by Borowicka (1943) for a finite layer for semi-infinite soil. Moment loading on rigid, rectangular footing on elastic half space is considered by Lee (1962). The case of eccentric loading on smooth, rigid, annular footing on elastic half space is investigated by Egorov et al. (1979). In the present work, moment-rotation relationship of soil-rigid footing system subjected to combination of moment and vertical load are evaluated considering soil modulus in unloading to be different from that in compression.

3. STATEMENT OF THE PROBLEM

A rigid footing resting on the surface of ground and subjected to vertical load, F prior to application of moment, M, is considered. The foundation- soil response is represented by a series of independent springs as in Winkler Model (Fig. 1a). The footing rotates through an angle $\theta$ due to the applied moment, M (Fig. 1b). The ground below one part of the footing undergoes compression while the other part undergoes unloading or stress reversal Figure 1(c). A modular ratio, $R_2$, is defined as $R_2 = k_u/k_c$, where $k_c$ and $k_u$ are moduli of subgrade reaction in compression due to moment and unloading respectively.

4. NON-LINEAR RESPONSE

Soil response with stress, $q$, and deformation, $\delta$, (Fig. 2) based on Kondner hyperbolic model is given by Eq. (1),

$$q = k_c \delta(1+k_c \delta/q_{ult})$$  \hspace{1cm} (1)

where $q_{ult}$ - the ultimate stress of soil, $k_c$ - the initial slope of the stress-displacement response of soil. It is necessary to consider the effect of vertical load prior to the application of moment, because in practice, the vertical load of the super-structure acts first and the moment loading gets applied subsequently. If the vertical load acts before the application of moment, the displacement has two components: uniform settlement due to central vertical load and rigid body rotation due to moment loading. The
response of the ground for the zone of soil which is already in compression due to the applied load prior to application of moment is as shown in Figure 2. The tangent modulus of subgrade reaction, \( k_{ct} \), is considered for the zone of compression due to moment. Linear stress-deformation relationship is assumed for unloading. The contact pressure distribution is shown in Figure 1(c).

\[ q = k_{ct} \delta \]

![Fig. 1: (a) Winkler Model of Soil-Structure Response (b) Vertical and Moment Loading on Footing (c) Contact Pressure due to Vertical and Moment Loading on Footing](image)

5. ANALYSIS-CIRCULAR FOOTING

A rigid circular footing of radius, \( R \), subjected to axial force, \( F \), and moment, \( M \), resting on the surface of ground is considered (Fig. 3a). The ground below one part of the footing undergoes compression while the other part undergoes unloading or stress reversal (Fig. 3b).

\[ q = k_{ct} \delta \]

![Unloading due to Moment](image)

The distance to the axis of rotation from the left edge of footing is \( x_0 \). The footing undergoes a deformation \( \delta^2 \) due to initial vertical stress, \( q \), prior to moment. The corresponding modulus is \( k_{ci} \). \( k_{ct} \) is related to \( k_{ci} \) as, Eq. 2:

\[ q = \frac{(k_{ci} \delta^2)}{(1 + k_{ci} \delta^2/q_{ult})} \]

\[ \delta^2 = \frac{(q/k_{ci})}{(1 - q/q_{ult})} \]

\[ \delta_* = \frac{\delta^2}{L} = \frac{q_*}{(1 - q_*)} \]

Where \( q_* = q/q_{ult} \) and \( \beta = q_{ult}/k_{ci} \)

The analysis consists of two steps, viz., force and moment equilibrium (Eqs 3 & 4) equations.

**Force equilibrium**

\[ f \int_0^R k_{ct} \delta\left(1 + \frac{k_{ct} \theta}{q_{ult}}(f_1)\right) f 2 dx \]

\[ + 2 \int_0^R x \int_0^{x_0} k_{ci} \theta \left(1 + \frac{k_{ci} \theta}{q_{ult}}(f_3)\right) f 2 dx \]

\[ - 2 \int_0^R x \int_0^{x_0} k_{ci} \theta (- f_3) f 2 dx = F \]  

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**Moment equilibrium**

\[ M \int_0^R \left(1 + \frac{k_{ct} \theta}{q_{ult}}(f_1)\right) f 2 dx \]

\[ + 2 \int_0^{x_0} x \int_0^{x_0} k_{ct} \theta \left(1 + \frac{k_{ct} \theta}{q_{ult}}(f_3)\right) f 2 dx \]

\[ - 2 \int_0^R \left(1 + \frac{k_{ct} \theta}{q_{ult}}(f_3)\right) f 2 dx = M \]  

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where \( f_1 = x + R - x_0 \), \( f_2 = R^2 - x^2 \), \( f_3 = R - x_0 - x \).

The integration is carried out numerically and the resulting expression for rotation, \( \theta \), is given by Eq. (5),

\[
\theta = M^* I_\theta
\]

where \( M^* = M/k_0 d^4 \) where \( k_0 \) is the initial tangent modulus in compression, \( d = 2R \) – the diameter of the footing, \( I_\theta \) – a function of \( X_0 = x/R \) and \( R_\theta \).

### 6. RESULTS AND DISCUSSION

The parametric values used in the present analyses: \( \beta = 0.005 \) to 0.1, \( R_\theta = 1 \) to 10 and \( q^* = 0 \) to 0.8. The effect of modular ratio, \( R_\theta \), on the moment-rotation relationships of rigid footings subjected to vertical and moment loading are obtained based on non-linear response of ground in compression. Madhav et al. (2009) presented the results for rectangular footing with the same assumptions. Hence, for comparison the same values have been used. Figure 4 presents the variation of normalized point of rotation \( X_0 = x/L \) for rectangular and circular footing with \( R_\theta \). The axis of rotation passes through the point which is at 0.21 times the length of rectangular and 0.42 times the diameter of the circular footing for \( R_\theta = 2 \). The axis of rotation shifts towards the zone of stress reduction with increasing inelasticity or \( R_\theta \) values. The rate at which the normalized point of rotation varies with increase in \( R_\theta \) is high for \( R_\theta < 10 \) for both rectangular and circular footings. The rate of decrease of \( X_0 \) with \( R_\theta \) decreases gradually for \( R_\theta > 10 \). Though the rate of shift of the axis of rotation is nearly the same for both types of footings for \( R_\theta < 10 \), the rate of decrease is somewhat faster for rectangular footing for higher \( R_\theta \). For example, at \( R_\theta = 20 \), \( X_0 \) shifts by about 63% of its length for rectangular footing, while it is only 53% of the diameter for circular footing.

Figure 5 depicts the variation of \( I_\theta \) with \( R_\theta \) for circular and rectangular footings. \( I_\theta \) decreases with increase in \( R_\theta \) for both footings. The results indicate lesser rotations for higher \( R_\theta \). The rate of decrease in \( I_\theta \) reduces with increase in \( R_\theta \). The values of \( I_\theta \) are 44.5 and 73.9 for \( R_\theta = 2 \), for rectangular and circular footing respectively. The reduction of \( I_\theta \) is about 85% of its value at \( R_\theta = 1 \) for circular footing and about 53% for rectangular footing for \( R_\theta = 25 \). Figure 6 shows the variation of ratio of \( I_\theta \) for a given \( R_\theta \) with that for \( R_\theta = 1 \), which is termed as \( I_{\theta 0} \). The variation of normalized \( I_\theta \) with \( R_\theta \) is nearly the same for circular and rectangular footings. Decrease of \( I_{\theta 0} \) with \( R_\theta \) is very high for \( R_\theta < 6 \) for both types of footings. The rate of decrease in normalized \( I_{\theta 0} \) decreases with increase in \( R_\theta \).

The moment-rotation relationship for circular footing for different \( R_\theta \) based on nonlinear analysis for \( \beta = (q_{ult} / K R) \) of 0.005 and \( q^* = 0.2 \) is shown in Figure 8. For a particular applied moment, rotation decreases markedly with increase in \( R_\theta \). The magnitude of moment increases for any rotation with increase in \( \beta \), though the rate of increase with \( \beta \) gets reduced with further increase of \( \beta \) (linear response). The effect of axial load prior to application of moment loading on moment-rotation relationship for
circular footing for $\beta = 0.005$ and $R_k = 2$ is shown in Figure 9. Moment-rotation relationship in case of nonlinear response is affected by the axial load applied before moment loading. As a result, rotation increases for a particular applied moment with increased vertical pressure prior to the moment application due lower values of tangent modulus in compression leading to higher modular ratio.

7. CONCLUSIONS

A new approach considering inelastic response of ground to combined vertical and moment loading of rigid circular footings based on non-linear response is presented. The inelastic response characterized by the stiffness ratio, $R_k$, the ratio of subgrade moduli in unloading to that in compression) has very significant effect on the overall response of circular footing to applied moment loading.

1. The axis of rotation shifts towards the unloading side from the center of the contact area with increase of $R_k$. The rate at which the axis of rotation gets shifted reduces gradually with increase of $R_k$.

2. The rotation influence coefficient, $I_\theta$, decreases with increase in $R_k$ implying that the conventional approach overestimates the rotation of footings due to moment loading. The rate of decrease of $I_\theta$ with $R_k$ reduces gradually.

3. The normalized values of $I_{\theta N}$ for any $R_k$ are nearly the same for rectangular and circular foundations.

4. The rotation of footing due to moment loading is more if an axial load acts prior to the application of moment loading.

REFERENCES


