**Computation of Sodium and Chloride Migration Through Layered Soil Media Using FDM**

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**ABSTRACT**

Finite difference method (FDM) has been adapted herein to analyze the performance of fully saturated clay barrier by using 1-D contaminant transport model to predict migration of two typical contaminant species, namely, sodium and chloride from the leachate of municipal waste landfill. The constructed clay liner media compacted in different lifts or the natural clay deposit beneath the landfill is heterogeneous in nature. This type of heterogeneity has been solved easily by idealizing the hydrostratigraphy of the barrier media as being horizontally layered with the soil properties being the same at any horizontal location within the layer. The program CONTAMINATE-1D LAYER has been developed to analyze the model with the newly developed solution technique simulating the parametric values those are evaluated and used by several other investigators previously. The concentration profiles have been illustrated in this paper. The design charts for liners are developed to facilitate the practicing engineers.

1. **INTRODUCTION**

The soil barrier beneath the landfill used in the disposal facility to separate the waste from the ground water system is generally heterogeneous in nature. The evaluation of the performance of the heterogeneous liner is complicated one. This type of heterogeneity has been solved easily by layering the hydrostratigraphy of the barrier with the soil properties being the same at any horizontal location within the layer. Two contaminant species, namely sodium and chloride which are mostly available in municipal waste landfill, are selected in this study. The one-dimensional steady flow maintaining continuity of flow condition has been assumed for each layer. The clay liner is underlain by a far more permeable thin stratum with ground water flow in the horizontal direction simulating the field situations. The permeable stratum is considered as confined by a lower impermeable boundary. The processes of advection, diffusion-dispersion and geochemical reactions have been considered for the design computation of liner in this study. The first order biochemical decay along with other processes have been considered for the preparation of concentration profiles. The constant surface concentration, Csur is assumed for any time for the development of concentration profiles and design charts. It is assumed that the pre-existing distribution of contaminant within the deposit, Cini is zero. The concentration profiles have been presented to illustrate the migration of contaminant species from the landfill leachate towards the aquifer beneath the clay liner.

2. **GOVERNING MODEL**

The 1-D partial differential equation along vertical downward z-direction applied to saturated homogeneous soil media, describing fate and transport of contaminants, the advection-dispersion-reaction-equation (ADRE) presented in Rowe and Booker (1985) is as follows:

\[
R \frac{\partial c}{\partial t} = D_z \frac{\partial^2 c}{\partial z^2} - v \frac{\partial c}{\partial z}
\]

Where, \( R \) = retardation Factor = \( 1 + \frac{\rho K_d}{\eta} \); \([\text{[-]}\) \( \rho \) = Dry density of the fully saturated subsurface medium; \([\text{ML}^{-2}]\); \( K_d \) = Partition (distribution) co-efficient of the contaminant species; \([\text{L}^3\text{M}^{-1}]\); \( \eta \) = porosity of the barrier; \([-\]); \( c \) = solute concentration of the contaminated site; \([\text{ML}^{-1}]\); \( t \) = time; \([\text{T}]\); \( z \) =Distance along the z-axis; \([\text{L}]\); \( D_z \) = Dispersion coeff.
along z-direction; \( [LT^{-1}] \); \( v \) = linearised seepage velocity along the z-axis; \( [LT^{-1}] \).

For contaminant species which undergo first-order decay, the rate of reduction of concentration is proportional to the current concentration and mentioned herein below:

$$\frac{\partial c}{\partial t} = -\lambda c$$  \hspace{1cm} (3)

Where, \( \lambda \) = first order decay constant; \( [T^{-1}] \).

Combining these two expressions (Eq.1&3) and extending it to include the effects of layering of the soil beneath the landfill the governing model to analyze the migration of contaminants in each of the homogeneous layer is as follows:

$$R \frac{\partial c}{\partial t} = D_z \frac{\partial^2 c}{\partial z^2} - v \frac{\partial c}{\partial z} - R\lambda c$$  \hspace{1cm} (4)

3. FINITE DIFFERENCE FORMULATION

In the model mentioned in the earlier section (Eq. 4), the value of the concentration is dependent on time as well as space. Again this equation contains the term involving the variable concentration, \( (c) \); its derivative w. r. t. time, \( \left( \frac{\partial c}{\partial t} \right) \); and derivative of concentration w. r. t. space direction z \( \left( \frac{\partial c}{\partial z} \right) \). The governing model (Eq. 4) is nonlinear in nature. Furthermore, the model is single degree and second order equation of parabolic type. Since the analytical solution technique to solve this model is more complex, simple FDM has been used in this study. In the present study, a two dimensional domain has been assumed as time \( t \) and depth \( z \) as thickness of liner increasing in the downward vertical direction. This is simulated for 1-D contaminant migration flow describing concentration of the contaminant species at different time at various depth of the liner. This formulation is made by explicit method. The domain is split into regular rectangular grids or meshes with \( k \) as time step and \( h \) as the spatial size in the z-direction vertically towards the aquifer.

Here, \( z = ih \); and \( t = jk \);  \hspace{1cm} (5-6)

Where, each of \( i, j = 0,1,2,3, \ldots \ldots \ldots \)

In the present study, solution of the partial differential equation has been done by using a forward difference scheme for single order derivative of concentration w. r. t. space, \( \left( \frac{\partial c}{\partial z} \right) \); and second order derivative of concentration w. r. t. space

$$\left( \frac{\partial^2 c}{\partial z^2} \right)$$ for better computation from the point of view of accuracy and stability (Jain 1991). The finite difference scheme of this model applied in each clayey layer of the barrier is as follows.

$$c[i][j] = \left( \frac{1}{p} \right) \left( Qc[j-1][i-1] + Sc[j-1][i] + Tc[j-1][i+1] \right)$$  \hspace{1cm} (7)

Where, \( R = 1 + \frac{\rho K_d}{\eta} \); \( P = \frac{R}{k} \); \( Q = - \frac{D}{h^2} - \frac{v}{2h} \);

\( S = \frac{2D}{h^2} - \frac{R}{k} + R\lambda \); \( T = \frac{v}{2h} + \frac{D}{h^2} \);

(8-12)

Sum of fitting parameters = \( P + Q + S + T = R\lambda \)  \hspace{1cm} (13)

The fitting parameters \( P, Q, S \) and \( T \) in the (Eq. 7) are used based on the sorption potential, \( \rho K_d \), porosity of the liner media, \( \eta \), dispersion coefficient, \( D \), linearised seepage velocity, \( v \), time step, \( k \) and spatial size, \( h \). The fitting parameters are so arranged that the sum of the four is zero indicating consideration of non-decaying type of contaminant solute in this study. This will also help in executing the program.

4. PROBLEM DEFINITION AND ASSUMPTION

The problem geometry of this present study is illustrated in Figure 1. The clay liner system is assumed to be segmented
in three homogeneous clayey layers underlying a thin permeable sandy layer each layer of 1.000 m thick. The transport of the pollutants, sodium and chloride, is assumed vertically downward from landfill towards the aquifer. The parametric values of contaminant transport corresponding to each layer including the aquifer have been taken from Rowe et al. (2004) and presented in Figure 1 and Table 1. The modified 1-D form of the advection-dispersion equation for reactive solute transport in saturated porous media is considered here, it maintains Fick’s first and second laws. The contaminant flow is in steady state. The sorption process is assumed as linear and reversible so that the mass of contaminant removed from solution is proportional to the concentration in the solution.

5. BOUNDARY CONDITIONS

The boundary conditions assumed to analyse the model using proposed finite difference scheme are as follows:

The concentration, \( c(z, t) \) at any depth, \( z \) beneath the surface of a barrier subjected to a maximum depth of 3.00 m and at any time, \( t \) and constant surface concentration \( c_0 \). The initial and other boundary conditions are as follows:

\[
\begin{align*}
\text{at } z = 0 : & \quad c(z, 0) = c_0 \\
\text{at } t > 0 : & \quad c(z, t) = 0 \text{ at } t > 0 \text{ and } z > 3.00 \text{ m} \\
\end{align*}
\]

Within the domain of the analysis of study, for any pair of adjacent layers \( i \)-th and \((i+1)\)-th, the steady continuity of contaminant flow has been assumed. So,

\[
\eta_i v_i = \eta_{i+1} v_{i+1}
\]

and the concentration at bottom of any layer is same as the concentration at top of the just next layer.

\[
c_i(z = z_i + z_i + ... z_i, t) = c_{i+1}(z = z_i + z_i + ... z_i, t)
\]

5. RESULTS AND DISCUSSION

The contaminant transport in a saturated soil media depends mainly on the type of contaminant and the hydrogeologic system of the landfill site. In the present study, the numerical model presented in the previous section has been used to simulate the transport of sodium and chloride ion from municipal waste landfill through layered soil media for specific field conditions. The porosity of three layers of liner has been considered as 0.30, 0.34 and 0.38 from the top to the bottom. The effects of radioactive and other decays are neglected for design calculations. The value of first order decay constant for biochemical degradation of chloride has been assumed as 0.065 per annum and for sodium species, it is considered as zero. The effect of first order decay constant is considered in concentration profile computations in this study. The linerised seepage velocity, \( v \) and advective velocity, \( v \), are assumed as 0.020 m/a and 0.006 m/a respectively at the first layer of the liner media. The advective velocity at these three different layers of liner is fixed for maintaining continuity of flow. The thickness of each layer of liner is 1.000 m. A thin 1.000 m thick permeable sandy stratum of porosity 0.40 is underlain beneath the liner.

6. CONCENTRATION PROFILES

Figure 2 presents concentration profile of non-decaying (\( \lambda = 0 \)) reactive species \( R \) as 1.600 to 1.474), sodium while Figure 3 illustrate concentration profile of decaying (= 0.065 per annum) non-reactive species \( R = 1.000 \), chloride for the instant of 5, 10, 20 and 40 years after the

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**Table 1: Parameters of Sodium and Chloride in Different Layers of Liner and Aquifer**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Sodium</th>
<th>Chloride</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dispersion coefficient in each layer</td>
<td>0.015</td>
<td>0.020</td>
</tr>
<tr>
<td>Sorption Potential ( \rho K_d )</td>
<td>0.180</td>
<td>0.000</td>
</tr>
<tr>
<td>Retardation Factor R for different layers and aquifer:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a) at the 1st layer</td>
<td>1.600</td>
<td>1.000</td>
</tr>
<tr>
<td>(b) at the 2nd layer</td>
<td>1.529</td>
<td>1.000</td>
</tr>
<tr>
<td>(c) at the 3rd layer</td>
<td>1.474</td>
<td>1.000</td>
</tr>
<tr>
<td>(d) at the base aquifer layer</td>
<td>1.450</td>
<td>1.000</td>
</tr>
<tr>
<td>( \lambda ) order bio-chemical decay constant</td>
<td>0.000</td>
<td>0.065</td>
</tr>
</tbody>
</table>

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**Fig. 2: Concentration Profiles of Sodium**

**Fig. 3: Concentration Profiles of Chloride**
time of peak source concentration. It presents the actual migration of pollutant which differs from seepage front. This is due to the effect of atomic weight of the species and decay.

7. DESIGN CHARTS OF SOIL LINERS

This section presents the design charts for earthen liners based on the proposed solution technique using parametric values of these three layers. Figures 4 and 5 present the design charts of liner for the contaminant species of sodium and chloride respectively. Each of these figures illustrate the set of curves for breakthrough time vs. liner thickness for 5%, 20%, 40%, 50%, 65% and 90% exit concentration ratio for particular contaminant species. These are made for different combination of dispersion coefficients, sorption potential, retardation factor as illustrated in Table 1. The linearised seepage velocity, \( v \) in different layers are presented in Figure 1 simulating the field situations. It is revealed from the charts that the required liner thickness for specific base concentration level varies with contaminant type. Depending on the presence of contaminants in the waste containment site the liner thickness should be provided accordingly. The trends of the set of curves are similar to that developed by Acar & Haider (1990). Similarly, the developed computer program CONTAMINATE-1D LAYER may be used to prepare design charts for specific contaminants for given hydrogeologic conditions to facilitate the field engineers.

8. SUMMARY AND CONCLUSIONS

A computer program CONTAMINATE-1D LAYER based on FDM has been developed for the analysis of sodium and chloride migration through three saturated soil layers of finite depth over a thin sandy aquifer layer.

Based on the analysis and discussion presented herein the following conclusions may be drawn:

1. The heterogeneous properties of liner media of total thickness can be analysed with a number of layers easily. The physical properties of the soil are assumed similar and considered homogeneous within each layer segment.

2. For a particular thickness of liner under certain hydrogeologic conditions of the site, the breakthrough time increases with increase in permissible base concentration and vice-versa.

3. For a given breakthrough time, the liner thickness increases with decrease in base concentration level, vice-versa, irrespective of contaminant species.

4. The concentration profiles of pollutant are different from species to species and depend on atomic weight and decay.

5. The computer program CONTAMINATE-1D LAYER developed based on FDM may be used for liner design for different field conditions.

REFERENCES


