Earthquake Wave Energy Flow and Distribution

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ABSTRACT

The mathematical modelling for earthquake wave energy flow from the earthquake source travel along the propagation path and finally reached to site leading to relative response of the structure. The total energy input into a structure exerted by an earthquake depends on the total mass and on the fundamental natural period of the structure. When the system is subjected to strong earthquake shaking, part of the incident wave energy is dissipated by scattering from the foundation and by deformation of soil along with isolator, and the rest is transmitted into the building. The equation of motion for the system are derived including non linear geometry and soil behaviour, coupling of soil acceleration with the rocking and horizontal translation, and the effect of gravity forces. For the building, the total input earthquake energy is calculated.

1. INTRODUCTION

Mathematical modelling essentially consists of translating real world problems into mathematical problems, solving the mathematical problems and then interpreting these solutions in the language of the real world.

Earthquake

Earthquakes are phenomena that result from the sudden release of stress in rocks that radiate seismic waves. At the Earth’s surface, earthquakes may manifest themselves by a shaking or displacement of the ground and sometimes tsunamis, which may lead to loss of life and destruction of property.

Generation of Waves

When earthquake occurs, seismic waves radiate away from the source and travel rapidly through the earth crust cause shaking of ground surface. The strength and duration of shaking depends upon the size and location of earthquake and characteristics of site. Soil deposits tend to act as “filters” to seismic waves by attenuating motion at certain frequencies and amplifying it at others.

Earthquake Motion Characterization

The earthquake motion is characterised by amplitude, frequency content and duration of motion. The motion parameter may be acceleration, velocity, or displacement. The measure of a ground motion is the peak horizontal acceleration (PHA) which has natural relationship to inertial forces, peak horizontal velocity (PHV) characterised ground motion amplitude accurately at intermediate frequencies and peak displacement associated with the lower frequency components of the earthquake motion.

2. EARTHQUAKE WAVE ENERGY FLOW AND DISTRIBUTION

The various stages considered for earthquake wave energy flow and distribution are: Prior to an earthquake the potential energy is stored in the rocks at the source, due to earthquake the energy losses are heat, friction, crushing of rocks and work against gravity forces; the elastic energy radiated in form of waves the energy losses are anelastic attenuation, scattering and geometric spreading; wave energy arriving towards the site the energy losses are non-linear response of shallow sediments and of soil in “free-field”; wave energy incident upon structural foundation the energy losses are scattering and diffraction of incident waves from the building foundation; response of soil with geosynthetics-foundation-structure systems the energy losses are non-linear response of soil due to soil-structure interaction, non-linear response of soil due to geosynthetic-soil-structure interaction and radiation damping; energy available to excite the structure the energy losses are hysteretic response of the structure and damping mechanism in the structure; the remaining energy is responsible for relative response of the structure.

Zahrah and Hall (1984) evaluated the non-linear response of simple structures and the damage potential of an earthquake ground motion, as measured in terms of the
amount of energy imparted to a structure, the amount of energy dissipated by inelastic deformation and by damping, as well as by assessment of the displacement ductility of the structure and the number of yielding excursions and reversals experienced during the excitation. Uang and Bertero (1988, 1990) discussed the derivation of the two “energy equations” (absolute and relative), and showed that the maximum values of the absolute and relative energy input, $E_i$ for any given constant displacement ductility ratio are very close in the period range of practical interest for earthquake-resistant design of buildings (0.3 to 5.0 seconds).

3. MODEL

Hao (2002) and Trifunac (2001) used the idealised mathematical model to describe the energy flow through a soil mass. Fig.1 show mathematical model to describe the energy flow through a soil-geosynthetics-foundation-structure system. In this model building is represented by an equivalent single-degree-of-freedom (SDOF) oscillator founded on a rigid embedded rectangular foundation (Takewaki 2005). The soil has shear modulus, $G$, shear wave velocity, $V_s$, Poisson’s ratio, $\nu$, and mass density $\rho_s$. The geosynthetic has spring stiffness, $k_g$, and the viscous damper has damping constant, $c_g$, and mass density is $\rho_g$, and $h$ is the depth of half-space below the foundation. The oscillator has only one degree-of-freedom with respect to the foundation $\theta^{ref}$. The mass of the oscillator is, $m_b$. It has height $H$ and radius of gyration, $r_b$. The oscillator is connected to foundation at point O through a rotational spring and a viscous damper. The spring has stiffness, $K_b$, and the viscous damper has damping constant, $C_b$. The stiffness is chosen such that the natural period of oscillator, $T$, is equal to the corresponding fixed-base period of the fundamental mode of the building. Assuming that the equivalent SDOF oscillator has same mass per unit length as the real building, assumed to deform in shear only, $H$ and $r_b$ are related to $H_{sb}$ and $W_{sb}$, the height and width of the real building (Todorovska and Trifunac, 1993), as $H = H_{sb}/\sqrt{3}$ and $r_b = W_{sb}/\sqrt{3}$.

The rectangular foundation has width $W_{sb}$, depth $D$, mass $m_r$, and mass moment of inertia $I_r$. To simplify the analysis, Hao (2002) and Trifunac, et al., 2001 assume that the stiffness of the soil in the vertical direction is infinite. The foundation has two-degree-of-freedom with respect to its centre of gravity (point CG): horizontal translation, $u$, and rotation, $\phi$ the foundation is surrounded by springs and dashpots, which model the relative forces caused by deformation developed in the soil. The top and bottom surface of geosynthetics surrounded by spring, dashpot which models the relative forces and roller model the horizontal translation. In figure 6, $k_r$ and $c_r$ are the stiffness and damping constants of horizontal springs and dashpots at the base of the foundation and $k_g$ and $c_g$ are the stiffness and damping constants of horizontal springs and dashpots at top as well as at bottom surface of geosynthetic representing the shear forces acting on the interface below the foundation; and $K_e$ and $C_e$ are the rotational stiffness and damping constant representing resisting moments in the half space. This soil-geosynthetic-foundation-oscillator system is subjected to horizontal and vertical excitations ($u$, and $v$).

4. EQUATIONS OF MOTION

The equation of motion of a linear SDOF system with viscous damping can be derived from the equilibrium of the forces and moments as

$$m \ddot{u} + c \dot{u} + k u = -m \ddot{\theta}(t) \quad (1)$$

From the equations of dynamic equilibrium of the forces in the horizontal and the vertical directions and all moments acting on the oscillator about point O, the interactive forces and the moment between the oscillator and foundation are

$$\Sigma F_x = 0 \Rightarrow \dot{f}_{x,b} = m_b \ddot{u}_b \quad (2)$$

$$\Sigma F_z = 0 \Rightarrow \dot{f}_{z,b} = -m_b (\ddot{\theta}_b - g) \quad (3)$$

$$\Sigma M_\theta = 0 \Rightarrow I_\theta (\dot{\phi} + \theta^{ref}) + m_b \ddot{\theta}_b H \cos(\phi + \theta^{ref}) + m_b (\dot{\theta}_b - g) H \sin(\phi + \theta^{ref}) + \frac{K_b}{\sqrt{3}} \theta^{ref} = 0 \quad (4)$$

Where, $I_\theta = m_b r_b^2 \frac{(\sqrt{3})^2}{H^2}$. 

![Fig. 1: Model of Soil-geosynthetics-foundation-structure interaction](image)
From the equations of dynamic equilibrium of all forces and moments acting on the foundation about CG, it follows that

\[
\begin{align*}
\Sigma F_x &= 0 \Rightarrow f_{x,a} = -m_f \ddot{u}_{CG} + f_{x,b} - f_{x,a} \\
\Sigma F_z &= 0 \Rightarrow f_{z,a} = -m_f \ddot{u}_{CG} + f_{z,b} - f_{z,a} \\
M_{CG} &= 0 \Rightarrow M_f = (f_{x,b} + f_{z,b}) \frac{D}{2} \sin \phi - \\
& - f_{x,a} \frac{D}{2} \cos \phi + M_{n,a} - I_r \theta - \Sigma \left( f_{x,i} \cos \phi \right) d_i + \\
& + \Sigma \left( f_{z,i} \sin \phi \right) d_i + \\
& + \left( f_{x,j} \cos \phi \right) h_j + \\
& + \left( f_{z,j} \sin \phi \right) h_j + \Sigma \left( f_{x,i} \cos \phi \right) h_j + \\
& + \Sigma \left( f_{z,i} \sin \phi \right) (h_j + \frac{D}{2})
\end{align*}
\]

(5)

(6)

(7)

where \(M_{CG}\) is the moment at the base of the foundation.

5. DERIVATION OF THE ENERGY EQUATIONS

To evaluate this work, we integrate all six equations (2), (3), (4), (5), (6), (7), with respect to the corresponding displacements. The total work done in the system is then computed by superposition of integrals of those equations. The energy balance equation for an SDOF structure can be written as (Uang and Bertero 1990).

\[
\int_0^t m \ddot{u} \, dt + \int_0^t c \dot{u} \, dt + \int_0^t f \, dt = -\int_0^t m \ddot{u} \, dt
\]

(9)

To simplify these energy formulae, we keep only the first order terms of the Taylor’s series expansion of sin and cosine functions of the angles \(\theta^{rel}\) and \(\varphi\) (and their linear combination), and eliminate the products of small angles and their derivatives. Then the above six equations give

\[
\begin{align*}
\int \left( m_ \dddot{u} + \frac{D}{2} \ddot{\theta} + H (\ddot{\varphi} + \dot{\theta^{rel}}) \right) \ddot{u} \, dt &= - \int m_ \dddot{u} \ddot{u} \, dt \\
\int \left( -m_ \dddot{v} + f_{v,b} \right) \ddot{v} \, dt &= - \int m_ \dddot{v} \ddot{v} \, dt \\
\int \left[ \left( u_{\theta^{rel}} + \frac{D}{2} \ddot{\theta} \right) H - m_ \dddot{u} \dot{H} \dot{\theta} + I_{c} \dddot{\varphi} + K_{\theta^{rel}} \right] \dddot{\varphi} \, dt &= - \int \left( m_ \dddot{u} \dot{H} \dot{\theta} + m_ \dddot{v} \dot{H} \dot{\theta} \right) \dot{\varphi} \, dt \\
\int \left( m_f \dddot{u} + f_{f,x} \right) u_{CG} \, dt + \int \left( \Sigma i f_{x,i} + \Sigma j f_{x,j} \right) u_{CG} \, dt &= - \int m_ \dddot{u} \ddot{u} \, dt \\
\int \left( -m_f \dddot{v} + f_{f,z} + f_{z,a} \right) v_{CG} \, dt &= - \int m_ \dddot{v} \ddot{v} \, dt
\end{align*}
\]

and

\[
\int \left( K_\theta \dddot{\theta} + C_\theta \dot{\theta} - f_{x,b} \ddot{\theta} + f_{x,f} \dddot{\theta} + f_{x,a} \right) \dddot{\theta} \, dt
\]

Next, we group the energy terms, according to their physical nature, into the following categories:

- \(E_k(t)\) = Kinetic energy
- \(E_p(t)\) = Potential energy of gravity forces
- \(E_d^{bld}(t)\) = Damping energy dissipated in the building
- \(E_d^{rel}(t)\) = Recoverable elastic strain energy in the building
- \(E_d^{soil}(t)\) = Energy dissipated by “dashpots” of the soil
- \(E_s(t)\) = Elastic strain energy in the soil
- \(E_{\gamma}(t)\) = Irrecoverable hysteretic energy in the soil
- \(E_{T}(t)\) = Total earthquake input energy (Takewaki 2005)

First, based on equations (10) through (14), the earthquake input energy is the sum of all the right hand side terms

\[
E_{T}(t) = - \int \left[ m_ \dddot{u} \dddot{u} + m_ \dddot{v} \dddot{v} + (m_ \dddot{u} \dot{H} \dot{\theta} + m_ \dddot{v} \dot{H} \dot{\theta} + m_ \dddot{u} \dot{H} \dot{\theta} + m_ \dddot{v} \dot{H} \dot{\theta} + m_ \dddot{u} \dot{H} \dot{\theta} + m_ \dddot{v} \dot{H} \dot{\theta}) \dddot{\varphi} \right] \dddot{\varphi} \, dt
\]

(16)

The kinetic energy associated with the absolute motion of two masses is

\[
E_k(t) = \int \left[ m_ \dddot{u} \dddot{u} + m_ \dddot{v} \dddot{v} + (m_ \dddot{u} \dot{H} \dot{\theta} + m_ \dddot{v} \dot{H} \dot{\theta} + m_ \dddot{u} \dot{H} \dot{\theta} + m_ \dddot{v} \dot{H} \dot{\theta} + m_ \dddot{u} \dot{H} \dot{\theta} + m_ \dddot{v} \dot{H} \dot{\theta}) \dddot{\varphi} \right] \dddot{\varphi} \, dt
\]

(17)

The potential energy associated with the gravity forces is

\[
E_p(t) = - \int \left[ m_ \dddot{u} \dddot{u} + m_ \dddot{v} \dddot{v} + (m_ \dddot{u} \dot{H} \dot{\theta} + m_ \dddot{v} \dot{H} \dot{\theta} + m_ \dddot{u} \dot{H} \dot{\theta} + m_ \dddot{v} \dot{H} \dot{\theta} + m_ \dddot{u} \dot{H} \dot{\theta} + m_ \dddot{v} \dot{H} \dot{\theta}) \dddot{\varphi} \right] \dddot{\varphi} \, dt
\]

(18)

The energy dissipated by viscous damping in the building can be calculated from

\[
E_d^{bld}(t) = \int C_\theta \left( \theta^{rel} \right)^2 \, dt
\]

(19)

The recoverable strain energy in the building

\[
E_d^{rel}(t) = \frac{1}{2} K_\theta \left( \theta^{rel} \right)^2
\]

(20)

For this problem, the building is assumed to deform in linear manner only, and the irrecoverable hysteretic energy in the building will be zero.
Then the energy dissipated by the dashpots in the soil is

\[ E_{D_s}^{\text{diss}}(t) = \int \left( \sum_{i=1}^{n} f_{R,i} \ddot{x}_i + \sum_{j=1}^{n} f_{R,j} \dot{\phi} \right) \ddot{x}_i \, dt + \int C_r \ddot{\phi}^2 \, dt + \int \left\{ \sum_{i=1}^{n} f_{R,i} \left( \ddot{x}_i - \frac{W_{bk} \phi}{k} \right) + \sum_{j=1}^{n} f_{R,j} \left( \dot{\phi} - \frac{W_{bk} \phi}{k} \right) \right\} \ddot{\phi} \, dt \]  

(21)

The energy dissipated by the yielding and by the recoverable strain energy of the soil can be obtained from which

\[ E_{S}^{\text{diss}}(t) = \int \left( \sum_{i=1}^{n} f_{s,i} \ddot{x}_i + \sum_{j=1}^{n} f_{s,j} \dot{\phi} \right) \ddot{x}_i \, dt + \int k_s \ddot{\phi}^2 \, dt + \int \left\{ \sum_{i=1}^{n} f_{s,i} \left( \ddot{x}_i - \frac{W_{bk} \phi}{k} \right) + \sum_{j=1}^{n} f_{s,j} \left( \dot{\phi} - \frac{W_{bk} \phi}{k} \right) \right\} \ddot{\phi} \, dt \]  

(22)

\[ k_{s,i}^{\text{diss}}(t) = \sum_{i=1}^{n} \left( \frac{f_{s,1}}{2k_s} \right)^2 + \sum_{j=1}^{n} \left( \frac{f_{s,2}}{2k_s} \right)^2 + \sum_{j=1}^{n} \left( \frac{f_{s,2}}{2k_{s,j}} \right)^2 + \frac{k_{s,j} \phi^2}{2} \]  

(23)

Where \( k_{s,i} \) and \( k_{s,j} \) are the initial stiffness coefficients of the inelastic soil and \( k_{s,j} \) is the initial stiffness of the geosynthetics.

Based on these energy “components,” the statement of energy balance of the system is then expressed as

\[ E_k(t) + E_p(t) + E_{D}^{\text{diss}}(t) + E_{S}^{\text{diss}}(t) + E_{D_s}^{\text{diss}}(t) + E_{S_s}^{\text{diss}}(t) = E_I(t) \]  

(24)

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