The paper pertains to the development of a semi-empirical method of analysis based on the conventional limiting equilibrium approach for predicting the uplift capacity of pile groups embedded in sand. Various pile and soil parameters such as length, diameter of the pile, group configuration, spacing of the piles, unit weight of the soil, angle of internal friction and the pile-soil interface friction angle having direct influence on the uplift capacity of the pile group are incorporated in the analysis. The predictive capability of the proposed model is verified with the model test results carried out on pile groups of different configuration and L/d ratio and embedded in medium dense soil. The predictions are found to be in good agreement with the measured values validating the developed method of analysis.

1. INTRODUCTION

Prediction of uplift capacity of single piles and especially of pile groups is one of the most interesting areas of research in geotechnical engineers. Some of the studies conducted on the behavior of single piles under uplift loads are due to Vesic (1970), Meyerhof (1973), Das & Seeley (1975), Ismael & Klym (1979), Das (1983), Rao & Venkatesh (1985), Chattopadhyay & Pise (1986), Alawneh et al. (1999), Ramasamy et al. (2004), Shanker et al. (2007). These studies are very helpful in understanding the behavior of piles under tensile loads and predicting the value of uplift capacity of single piles. But, the ultimate load capacity of a pile group is not necessarily the sum of the capacities of the individual piles within the group. The methods available to foundation engineers for predicting pile group capacities in tension are mainly of an empirical nature. Increasing use of group of piles to resist and sustain uplift loads requires accurate assessment of uplift resistance for safe and economical design of pile foundations. As such, literature on such studies on the uplift capacity of pile groups embedded in sand is scanty (Das et al. 1976, Chattopadhyay 1994, Patra & Pise 2003, Shanker et al. 2006). The theories to predict the uplift capacity of piles were developed mostly by extending the analysis of horizontal plate anchors under uplift loads assuming development of failure surfaces starting from the edges of the anchor. Based on experimental observation and test data, Meyerhof & Adams (1968) proposed a general theory of uplift resistance for a strip footing in soils with the assumption that soil mass having approximately truncated pyramidal shape is lifted up and for shallow footing the failure surface extends up to the ground surface. Das et al. (1976) and Chattopadhyay (1994) concluded based on model test results that the efficiency of a pile group varies with embedment length, number of piles in a group and spacing of piles in a group. As such, there is a need to develop methods to predict the uplift capacity of pile groups and such an analysis is presented in this study. The uplift resistance of pile groups depends on the several variables like group size, shape, spacing, embedment length to diameter ratio of piles, soil type and its density and soil-pile friction angle. Considering some of the above parameters a simplified analysis based on limiting equilibrium is proposed to predict the uplift capacity of the group of piles. Laboratory model tests on group of piles were conducted in medium dense sand under axial uplift loads at different pile spacing and with varying embedment length to diameter ratio. The predicted values of uplift capacity were compared with the model test results so obtained and with other published experimental data to check the validity of the developed method of analysis.

2. ANALYSIS

Figure 1 shows a typical general pile group (a 2×2 pile group is shown here) of length L embedded in soil. The
Uplift Capacity of Pile Groups Embedded in Sand

2.1 Failure surface

During uplift of the pile, an axisymmetric solid body of revolution of soil will also move with it, resulting in a curved surface, as suggested by Chattopadhyaya & Pise (1986) and Shanker et al (2007). In the present analysis the failure surface developed by Shanker et al (2007) is used to predict the uplift capacity of group of piles. The slope of the failure surface at a height \(z\) above the pile tip (Fig. 1) can be written as:

\[
\frac{dz}{dx} = \tan\left(90 - \left(\frac{45 + \phi / 2}{L}\right)z\right) \exp\left[\beta \left(1 - \left(\frac{z}{L}\right)^\gamma\right)\right]
\]

(Shanker et al. 2007) \quad (1)

Where \(\beta = (\lambda)^{- \frac{(\phi_{\text{max}} - \phi)}{2\delta}}\) \quad (2)

The maximum value of angle of friction (\(\phi_{\text{max}}\)) for practical purposes has been assumed to be 50°.

In order to determine the failure surface profile Eqn. (1) has to be integrated. As direct integration is complicated, the solution is conveniently obtained by the numerical method of integration.

\[
\text{(P}_{\text{mu}}) = N \int_{0}^{z_{\text{eff}}} \pi d \left(1 - \frac{z}{L}\right) \left(\frac{1}{M_1} + M\right)dz
\]

be equal to (1-sin\(\phi\)) and \(\theta = \tan^{-1}\left(\frac{dz}{dx}\right)\)

\[
M = (\cos \theta + K \sin \theta) \tan \phi
\]

Where \(K\) is the lateral earth pressure coefficient assumed to

2.2 Ultimate Uplift Capacity

The uplift capacity of the pile group problem can be analyzed considering two different cases depending upon the centre to centre spacing (s) of pile and maximum lateral extent of failure surface (\(X_G\)) on the ground surface as follows:

2.2.1 Case-1 (s \(\geq\) \(2X_G\))

For this case overlapping of failure surfaces does not take place and each pile in the group acts independently (Fig. 1). Hence, the net uplift capacity of the pile group is equal to the number of piles in the group (N) times the net uplift capacity of a single pile as follows,

\[
\text{P}_{\text{mu}}(\text{Group}) = N \times \text{P}_{\text{mu}}
\]

2.2.2 Case-2 (s < \(2X_G\))

For this case overlapping of failure surfaces takes place at a height of \(z_{\text{eff}}\) as shown in Figure 2. The net uplift capacity of the pile group can be evaluated by dividing the problem into two parts as follows.

\[
\text{(P}_{\text{mu}}) = N \int_{0}^{z_{\text{eff}}} \pi d \left(1 - \frac{z}{L}\right) \left(\frac{1}{M_1} + M\right)dz
\]

Part-1 (0 \(\leq\) \(z\) \(\leq\) \(z_{\text{eff}}\))

The net uplift capacity of the pile group in this region can be evaluated in a similar way as discussed in Case-1 by changing the limit of integration 0 to \(z_{\text{eff}}\) (Fig. 2).

\[
\text{(P}_{\text{mu}}) = N \int_{0}^{z_{\text{eff}}} \pi d \left(1 - \frac{z}{L}\right) \left(\frac{1}{M_1} + M\right)dz
\]

Part-2 (\(z_{\text{eff}}\) \(\leq\) \(z\) \(\leq\) \(L\))

In deriving the expression for the uplift capacity of pile group in this overlapped zone it is assumed that under limiting condition soil mass around the pile group fails as an inverted truncated solid body. In the limiting equilibrium
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condition, ultimate capacity of the pile is attained when the mobilized shear strength along the failure surface and the weights of the soil and piles balance the applied forces. A wedge of thickness $\Delta z$ at a height $z$ above the tip of the pile is considered; the wedge and its free body diagram are shown in Figure 2.

The mobilized shear resistance $\Delta T$ along the failure surface of length $\Delta L$, at limiting condition is,

$$\Delta T = \Delta R \tan \phi$$

Where $\Delta R$ = Normal force acting on the failure surface of the wedge

$$\Delta R = \Delta Q \cos \theta + K \Delta Q \sin \theta$$

(8)

![Pile Group with Overlapping of Failure Surfaces](image)

Fig. 2: Pile Group with Overlapping of Failure Surfaces

Where,

$$\Delta Q = \gamma (L - z - \frac{\Delta z}{2}) \Delta L \quad \text{and} \quad \theta = \tan^{-1} \left( \frac{dz}{dx} \right)$$

(9)

Substituting Eqn. 9 in Eqn. 8 we get,

$$\Delta R = \gamma \left( L - z - \frac{\Delta z}{2} \right) \left( \cos \theta + K \sin \theta \right) \frac{\Delta z}{\sin \theta}$$

(10)

Substituting Eqn. 10 in Eqn. 7, we get

$$\Delta T = \gamma \left( L - z - \frac{\Delta z}{2} \right) \left( \cos \theta + K \sin \theta \right) \frac{\Delta z \tan \phi}{\sin \theta}$$

(11)

Considering the vertical equilibrium of the wedge,

$$(P + \Delta P) - P + q \left[ (a + 2x_2)(b + 2x_3) - 4A_\Delta - N A_p \right] - q_1 \left[ (a + 2x_2 + 2\Delta x)(b + 2x_3 + 2\Delta x) \right] - g_0 \left[ (a + 2x_2 + 2\Delta x)(b + 2x_3 + 2\Delta x) \right] q_1 \left( -4A_\Delta - N A_p \right) - \Delta W - 2(a + b - 2d + 4p)\Delta T \sin \theta = 0$$

(12)

Where,

$$x_2 = x - \frac{d}{2} \quad \Delta W = (a + 2x_2)(b + 2x_3) + \Delta x(a + b + 4x_3) + 2\Delta x^2 - 4\Delta A \frac{\gamma \Delta A}{2}$$

$$\Delta A = (x_2 + \frac{d}{2})^2 (1 - \frac{\pi}{4}), \quad A_p = \frac{\pi d^2}{4}, \quad q = \gamma (L - z),$$

$$q_1 = q (L - z - \Delta z) \quad \text{and} \quad p = \frac{\pi}{2} (x_2 + \frac{d}{2}) $$

(13)

Substituting Eqn. 13 in Eqn. 12 and on simplification we get

$$\frac{dP}{dz} = 2\gamma (L - z)(a + b + 4x_2) \frac{dx}{dz} + \gamma NA_p + \left\{ 2(a + b - 2d) + 2\pi \left( x_2 + \frac{d}{2} \right) \right\} (L - z) M$$

(14)

Where, $M = (\cos \theta + K \sin \theta) \tan \phi$ and

$$\frac{dx}{dz} = \frac{1}{\tan \left( \tan^{-1} \left( \frac{45 + \phi}{2L} \right) \right) \exp \left( \beta \left( 1 - \left( \frac{z}{L} \right)^{\frac{1}{3}} \right) \right)}$$

On integration of Eqn. 14 in the limit $z_{eff} \leq z \leq L$ and deducting the self weight of the pile group yields an expression for the net uplift capacity of the pile ($P_{nu}$) is given by

$$P_{nu} = \int_{z_{eff}}^{L} \frac{dP}{dz} dz$$

$$= \int_{z_{eff}}^{L} 2\gamma L \left( 1 - \frac{z}{L} \right) \left( K_2 + 4x_2 \frac{dx}{dz} \right) dz + \left( K_2 - 2 + \frac{\pi}{2} \left( \frac{x_2 + \frac{d}{2}}{2} \right) \right) M$$

(15)

Where $K_2 = (a + b)/d$

Now, Total net uplift capacity of pile group is equal to

$$P_{nu} = (P_{nu1} + P_{nu2})$$

(16)

The above predictive model is validated by experimental studies made on model piles, which is detailed as follows.

3. RESULTS AND DISCUSSION

The theoretical values of the net uplift capacities for different group of piles considered for experimental studies were estimated using the proposed model by assuming different trial values of constant (C) in Eqn. 2. From several trials so made it is observed that at C value equal to 1.9 the predicted values are in very good agreement with the experimental results. Therefore, the same is used for further predictions. A quantitative comparative study was made to estimate the deviation of the predicted uplift capacity from the measured
one available in the literature (Shanker et al. 2007, Das et al. 1976, Patra & Pise 2003). It is observed that for 98% of the data (44 out of 45) the deviation was within 30% and for 82% of the data (37 out of 45) the error is even less than 20%. Thus the absolute relative errors between the predicted and measured values lie in general in a range which may be considered to be well within the range of experimental error and errors inherent to the models. However, for the sake of space and shortness these details are not presented here.

For better appreciation of the predicting capability of the proposed model a cumulative frequency table for the data corresponding to the absolute percentage of errors is presented in Table 1. From this table it is observed that for 85% of the data (63 data points out of 74 data points) the error is within the 30%.

**Table 1: Absolute Error Distribution of Predictions**

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Range of absolute error (%)</th>
<th>No. of data</th>
<th>Cumulative data</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0–5</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>2</td>
<td>5–10</td>
<td>14</td>
<td>31</td>
</tr>
<tr>
<td>3</td>
<td>10–15</td>
<td>8</td>
<td>39</td>
</tr>
<tr>
<td>4</td>
<td>15–20</td>
<td>11</td>
<td>50</td>
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<tr>
<td>5</td>
<td>20–25</td>
<td>5</td>
<td>55</td>
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<tr>
<td>6</td>
<td>25–30</td>
<td>8</td>
<td>63</td>
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<tr>
<td>7</td>
<td>30–35</td>
<td>1</td>
<td>64</td>
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<tr>
<td>8</td>
<td>35–40</td>
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<td>67</td>
</tr>
<tr>
<td>10</td>
<td>45–50</td>
<td>4</td>
<td>71</td>
</tr>
<tr>
<td>11</td>
<td>&gt;50</td>
<td>3</td>
<td>74</td>
</tr>
</tbody>
</table>

**4. CONCLUSIONS**

From the above study the following conclusions are drawn.

The ultimate uplift capacity of the pile group depends on the embedment length to diameter ratio of the pile, pile group configuration, center to center spacing of piles in a group. The proposed semi empirical model has the potential in predicting the uplift capacity of pile groups embedded in sand. The error margin between the predicted values and the experimentally observed values in general is well within 30%.

**REFERENCES**


**NOTATION**

The following symbols are used in this paper.

- \( a \) & \( b \): plan dimensions of the pile group
- \( b_1 \): depth of overlapping
- \( C \): dimensionless constant
- \( D_r \): relative density
- \( d \): pile diameter
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$K$  lateral earth pressure coefficient
$L$  embedded length of pile
$\lambda$ ratio of embedded length to diameter of pile (L/d)
$P_u$ ultimate uplift capacity of pile
$P_{nu}$ net ultimate uplift capacity of pile
$s$ centre to centre spacing of the piles
$\Delta W$ weight of the elemental strip
$W_g$ self weight of the pile group
$x$ lateral extent of the failure surface from pile axis
$x_1$ lateral extent of the failure surface from pile surface
$X_G$ lateral extent of the failure surface at ground surface
$\Delta z$ thickness of wedge element
$z_{eff}$ depth without overlapping
$\theta$ angle of failure surface with horizontal
$\phi$ angle of internal friction of the soil
$\delta$ pile-soil interface friction angle
$\gamma$ unit weight of the soil