ABSTRACT: Finite difference method has been adopted herein to solve 1-D contaminant transport model to predict the sodium and chloride migration through porous media in municipal waste landfill, considering both the short term and long term effects. Providing an effective engineered barrier, which will separate the waste from ground water, can minimize the potential contamination. Optimal design of waste disposal facilities requires an understanding of the fundamental mechanisms and the material properties in the appropriate chemical and hydraulic environment, as well as the availability of mathematical models. In the Finite difference technique, the velocity field is first determined within a hydrologic system, and these velocities are then used to calculate the rate of contaminant migration by solving equation. This technique is well suited for complex geometrics, complicated flow patterns heterogeneity and non-linearity. The computer program, CONTAMINATE has been developed to solve the 1-D migration equation. The efficacy of the developed model has been studied comparing the results available in literature.

1. INTRODUCTION

Geotechnical engineers are mostly involved in the analysis of sodium and chloride migration from the leachates produced in municipal waste landfill. This paper presents a technique for the analysis of a single solute in a layer of finite thickness of existing clay deposit or clay liner. This paper considers the combined effects of advection, diffusion-dispersion, geo-chemical retardation and first order biological and chemical decay under saturated condition in one model. Based on the parametric values evaluated by earlier researchers have been used for the execution of the programs and analysis of this study. This paper neglects the three components of first order decay due to radioactivity, biological decay and fluid withdrawal for sodium migration; But 0.065 value is assumed as first order decay constant due to biological and chemical decay. The constant surface concentration Csur is assumed for any time for the preparation of design charts. The concentration profiles have been presented assuming the value of height of leachate Hf as 1m simulating the field condition such that the migration of the contaminant towards the ground water beneath the clay deposit-liner from a finite quantity of pollutant in the landfill. The next section presents the governing equation of 1-D contaminant migration.

2. GOVERNING PARTIAL DIFFERENTIAL EQUATION

The key factors governing contaminant migration are advection, dispersion and geo-chemical and other chemical reactions. The one-dimension form of the governing equation applied to saturated homogeneous soil media conditions (Rowe & Booker 1985), the ADRE (Advection-dispersion-reaction-equation) is:

\[ R \frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial z^2} - v \frac{\partial c}{\partial z} \]  

Again, the rate of reduction of concentration of radioactive or biologically and chemically decaying solute species is proportional to the available concentration of that solute, considering first order decay such that

\[ R \frac{\partial c}{\partial t} = -\lambda c \]  

Combining the effect of first order decay on ADRE, the governing partial differential equation in this paper is

\[ R \frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial z^2} - v \frac{\partial c}{\partial z} - \lambda c \]  

Where,

\[ R = \text{Retardation Factor} = 1 + \frac{\rho K_d}{\eta} \]  
\[ \rho = \text{Dry density of the subsurface medium; } [\text{ML}^{-1}] \]  
\[ K_d = \text{Distribution co-efficient; } [\text{L}^3\text{M}^{-1}] \]  
\[ c = \text{solute concentration in the pore fluid of the liner; } [\text{ML}^{-3}] \]  
\[ t = \text{time; } [\text{T}] \]
Finite Difference Method for Computation of Sodium and Chloride Migration in Porous Media

$z$ = Distance along the respective Cartesian coordinate axis $Z$ respectively; [L].
$D$ = dispersion coefficient; [L$^2$T$^{-1}$].
$v$ = discharge velocity calculated by Darcy’s law
= (Co-efficient of hydraulic conductivity $k \times$ hydraulic gradient $i$); [LT$^{-1}$].
$v_a$ = advective velocity = (discharge velocity, $v$) \times (porosity, $\eta$); [LT$^{-1}$].
i = hydraulic gradient = (Difference of head, $\Delta h$) / (Length of medium, $\Delta l$).
$\lambda$ = the lumped first-order decay constant = sum of first-order decay constant due to biological decay, chemical decay, radioactivity, and fluid withdrawal if any; [T$^{-1}$].

Based on the above mentioned governing equation, finite difference formulation has been made.

### 3. FINITE DIFFERENCE FORMULATION

Since the analytical solution technique is more complex, simple Finite Difference Method (FDM) has been used to solve this model. In this paper, a two dimensional domain describing time $t$ and spatial direction $z$ as depth has been considered with grid sizes $k$ and $h$ in the time and depth directions respectively (Fig. 1).

![Grid Points in the Finite Difference Method for 1-D Contaminant Migration Problem](image)

Using the forward difference scheme for single order derivative of concentration w.r.t. time and central difference scheme both for single order derivative of concentration w.r.t. space and second order derivative of concentration w.r.t. space for better computation from the point of view of accuracy and stability (Jain 1991), the general form of 1-Dimensional model in this study can be replaced by finite difference form as follows.

$$
R \frac{c[j+1][i] - c[j][i]}{k} = D \frac{c[j][i+1] - 2c[j][i] + c[j][i-1]}{h^2} - \frac{c[j][i+1] - c[j][i-1]}{2h} - \lambda c[j][i]
$$

$$
(4)
$$

where, $j$ and $i$ are the indices two dimensional concentration array in the time and space directions respectively.

Therefore, finite difference form of 1-D contaminant migration relation through soil is to be applied for computer program as follows:

$$
c[j][i] = \left(-\frac{1}{P}\right)\left((Qc[j+1][i-1] + Sc[j-1][i]) + Tc[j-1][i+1]\right)
$$

Where, $P = \frac{R}{k}$; $Q = -\frac{D}{h^2} + \frac{v}{2h}$; $S = \frac{2D}{h^2} + \frac{R}{k} + \lambda$; $T = \frac{2h}{h^2} - \frac{D}{h^2}$; $\lambda$ = the lumped first-order decay constant = sum of first-order decay constant due to biological decay, chemical decay, radioactivity, and fluid withdrawal if any; [T$^{-1}$].

Sum of fitting parameters = $P + Q + S + T$;

(10)

A set of programs has been developed based on the above mentioned finite difference formulation. The fitting parameters in this study (equations 6–9) are used based on the retardation factor, $R$, dispersion coefficient, $D$, discharge velocity, $v$ and the distance of the grid in the space and time directions $h$ and $k$ respectively. The sum of the fitting parameters $P$, $Q$, $S$ and $T$ (equation 10) showing zero indicates consideration of non-decaying type of contaminant solute in this study, otherwise decaying type of solute in this study. This will also help to execute the programs. On the basis of outputs of the above computer programs simulated with the municipal waste landfill, the following results have been drawn.

### 4. RESULTS AND DISCUSSION

The numerical model presented in this paper has been simulated the sodium and chloride migration through the clay liner from the municipal waste landfill. In this section, Figs. 2 to 4 present the concentration profiles of sodium, chloride (neglecting decay) and chloride (considering decay) at different depths of liner commencing from the bottom most surface of the landfill. The parametric values have been taken from Rowe et al. (2004). Whereas Figures 5 and 6 illustrate the design charts of sodium and chloride migration through compacted clay liner.

#### 4.1 Concentration Profiles

Figure 2 presents concentration profile of non-decaying (first order decay constant $\lambda$ or $L$ as zero) reactive ($R = 1.36$) of sodium. Figure 3 illustrates concentration profile of non-reactive ($R = 1$) solute chloride (neglecting decay, first order decay constant $\lambda$ or $L$ as zero), whereas Figure 4 shows concentration profile of non-reactive ($R = 1$) solute chloride (considering decay, first order decay constant $\lambda$ or $L$ as 0.065).
From the above three graphs, it is noted that as time passes the concentration in the liner increases; chloride passes much faster than sodium species; since decay indicates mass loss of a particular contaminant, height of leachate ($H_f$) is essential for analysis of migration for decaying solutes.

Fig. 2: Concentration Profile of Sodium Migration at Different Depths of Compacted Clay Liner for Various Breakthrough Time

Fig. 3: Concentration Profile of Chloride Migration at Different Depths of Compacted Clay Liner for Various Breakthrough Time (neglecting decay)

Fig. 4: Concentration Profile of Chloride Migration at Different Depths of Compacted Clay Liner for Various Breakthrough Time (considering biological and/or chemical decay)

4.2 Design Charts

Figures 5 and 6 present the design charts of liner thickness vs breakthrough time for sodium and chloride migration respectively for a given set of performance criteria. These sets of curves are drawn based on fixed surface concentration ($C_{sur} = 100$ unit) and zero initial concentration ($C_{ini} = 0$ unit) for the consideration of non-decaying solute nature executing the set of programs CONTAMINATE.
5. CONCLUSIONS

Based on the study in this paper as well as graphs of concentration profiles and design charts of sodium and chloride, the following may be concluded:

1. If the time increases the contaminant species passes through a larger depth of the barrier.
2. Chloride species passes comparatively faster than that of sodium species through the compacted clay liner and this may be due to the higher molecular weight of chloride ion than that of sodium ion.
3. Height of leachate consideration is essential for analysing the migration of pollutant especially for decaying contaminant species, since it includes the implication of the mass available in the landfill leachate.
4. Service life of a liner may be increased up to a certain limit by increasing the thickness of the liner.

REFERENCES


