Solutions of Numerical Method

February 24, 2016

Q - 1. What is the order of convergence in bisection and Newton-Raphson method?

A - 1. Order of convergence in bisection method is linear.
Order of convergence of in newton raphson method is quadratic.

Q - 2. Find the positive root of equation $x^3 - x - 4 = 0$ correct to two decimal places using bisection method

A - 2. $x^3 - x - 4 = 0$
$f(1) = -4$
$f(2) = 2$
so, the root lies between 1 and 2

$$x_1 = 1/2(1 + 2) = 1.5$$

Step1: $x_1 = 1.5$, $f(x_1) = -2.125$
Step2: $x_2 = 1.75$, $f(x_2) = -0.390625$
Step3: $x_3 = 1.875$, $f(x_3) = 0.7167$
Step4: $x_4 = 1.8125$, $f(x_4) = 0.141$
Step5: $x_5 = 1.78125$, $f(x_5) = -0.129$
Approximate value is 1.78125

Q - 3. Define Characteristic equation, Eigen values and Eigen vectors associated with square matrix

A - 3. Characteristic Equation: If A is a square matrix of order n with elements a_{ij}, we can find a column matrix X and a constant λ such that AX=λX or AX-λIX=0. On expansion, it gives an n_{th} degree equation in λ, called the characteristics equation of the matrix A. Its roots λ_i (i=1,2,...n) are called the eigen values. And corresponding to each eigen value will have a non-zero solution

$$X = [x_1, x_2, x_3, ...x_n]'$$

which is known as the eigen vector.

Q - 4. Solve the following system of equations using Gauss Jordan method

$$5x + 4y = 15$$

$$3x + 7y = 12$$

A - 4. Gauss Jordan Method:

$$5x + 4y = 15 \tag{1}$$

$$3x + 7y = 12 \tag{2}$$

Multiplying 1^{st} equation with 3

$$15x + 12y = 45$$

Multiplying 2^{nd} equation with 5

$$15x + 35y = 60$$

Subtracting both the equations

$$y = \frac{15}{23} = 0.6521$$

Now, putting the value of y in any equation and we get,

$$x = \frac{285}{23} = 12.391$$

Q - 5. Using Newton Raphson formula, show that square root of N=AB in given by $\sqrt{N} = \frac{S}{4} + \frac{N}{S}$ where $S = A + B$

A - 5. Let,

$$x = \sqrt{N}$$

$$x^2 - N = 0$$

$$f(x) = x^2 - N$$

$$f'(x) = 2x$$

By Newton-Raphson,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{{x_n}^2 - N}{2x_n}$$

$$x_{n+1} = x_n - \frac{x_n}{2} - \frac{N}{2x_n}$$

$$x_{n+1} = \frac{x_n}{2} + \frac{N}{2x_n}$$

Suppose,

$$x_n = \frac{A+B}{2}$$

So,

$$x_{n+1} = \frac{A+B}{2*2} + \frac{N}{A+B} = \frac{S}{4} + \frac{N}{S}$$

$$\therefore S = A + B$$

Q - 6. Solve the system of equation by Gauss Siedel iteration method

$$4x + y + z = 4$$

$$x + 4y - 2z = 4$$

$$3x + 2y - 4z = 6$$

A - 6. Gauss Seidel Method:

$$4x + y + z = 4$$

$$x + 4y - 2z = 4$$

$$3x + 2y - 4z = 6$$

Now,

$$x = \frac{1}{4}(4 - y - z) \tag{3}$$

$$y = \frac{1}{4}(4 + 2z - x) \tag{4}$$

$$z = -\frac{1}{4}(3x + 2y - 6) \tag{5}$$

Put $y = 0, z = 0$ in (1)

$$x = 1$$

Put $x = 1, z = 0$ in (2)

$$y = \frac{3}{4} = 0.75$$

Put the values of x and y in (3)

$$z = \frac{-3}{8} = -0.375$$

Second iteration:

Put $y = \frac{3}{4}$ and $z = \frac{-3}{8}$ in (1) we get,

$$x = \frac{29}{32} = 0.90625$$

put, $x = 0.90625$ and $z = -0.375$ in (2) we get,

$$y = 0.5859$$

put, $x = 0.90625$ and $y = 0.5859$ in (3) we get,

$$z = -0.5273$$

Third iteration:

put, $y = 0.5859$ and $z = -0.5273$ in (1) we get,

$$x = 0.98535$$

put, $x = 0.98535$ and $z = -0.5273$ in (2) we get,

$$y = 0.4900$$

put, $x = 0.98535$ and y=0.4900 in (3) we get,

$$z = -0.5159$$

Fourth iteration:

put, $y = 0.4900$ and $z = -0.5159$ in (1) we get,

$$x = 1.006475$$

put, $x = 1.006475$ and $z = -0.5159$ in (2) we get,

$$y = 0.4904$$

put, $x = 1.006475$ and $y = 0.4904$ in (3) we get,

$$z = -0.4999$$

Q - 7. Find roots by Newton-Raphson method of following:

$$3x^2 - 12x + 2 = 0$$

A - 7. $f(x) = 3x^2 - 12x + 2$

At $x = 3$

$$f(3) = -7$$

At $x = 4$

$$f(4) = 2$$

So, Roots between 3 and 4

$$f'(x) = 6x - 12$$

Now, by using newton-raphson method

$$x_{n+1} = x_n - \frac{f(x)}{f'(x)}$$

take $x_0 = 3.5$

$$x_1 = x_0 - \frac{3{x_0}^2 - 12x_0 + 2}{6x_0 - 12}$$

1^{st} iteration:

$$x_1 = 0.5 - \frac{3(3.5)^2 - 12(3.5) + 2}{6(3.5) - 12}$$

$$x_1 = 3.8611$$

2^{nd} iteration:

$$x_2 = 3.8611 - \frac{3(3.8611)^2 - 12(3.8611) + 2}{6(3.8611) - 12}$$

$$x_2 = 3.8260$$

3^{rd} iteration:

$$x_3 = 3.8260 - \frac{3(3.8260)^2 - 12(3.8260) + 2}{6(3.8260) - 12}$$

$$x_3 = 3.8257$$

4^{th} iteration:

$$x_4 = 3.8257 - \frac{3(3.8257)^2 - 12(3.8257) + 2}{6(3.8257) - 12}$$

$$x_4 = 3.8257$$

So, real root of equation is 3.8257