SEMINAR

CONTENTS

STRAIN – DISPLACEMENT RELATIONS

The displacement of a point within the solid body is a function of the three coordinate directions. The displacement components of a point in x, y and z directions are u, v and w respectively.

u = f (x, y, z), v = g (x, y, z) and w = h (x, y, z) (1.1)

The objective of FEM is to determine the equations corresponding to f (x, y, z), g (x, y, z) and h (x, y, z). The finite element approximations are continuous, piecewise smooth equations defined over the individual elements. The element equations depend on the type element used to solve the problem.

The six independent stresses are:

$$\sigma = [\sigma_{x,}\ \sigma_{y,}\ \sigma_{z,}\ \sigma_{xy,}\ \sigma_{yz,}\ \sigma_{zx}]^T \quad (1.2)$$

Where

$\sigma_{x,}\ \sigma_{y,}\ \sigma_z$ = Normal stresses

$\sigma_{xy},\sigma_{\ yz,}\ \sigma_{zx}$ = Shear stresses

The corresponding strains are:

$$\varepsilon = [\varepsilon_{x,}\ \varepsilon_{y,}\ \varepsilon_{z,}\ \gamma_{xy,}\ \gamma_{yz,}\ \gamma_{zx}]^T \quad (1.3)$$

Where

$\varepsilon_{x,}\ \varepsilon_{y,}\ \varepsilon_z =$ Normal strains

$\varepsilon_{xy,}\ \varepsilon_{yz,}\ \varepsilon_{zx=}$ Shear strains

The Strain- displacement relations are:

$$\varepsilon = [\frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial w}{\partial z}, \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right), (\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x})]^T$$

Example 1. If a displacement field is described by

u = (-$x^2 + 2y^2 + 6xy)10^{-4}$ $and\ \ v = (3x + 6y - y^2)10^{-4}$

Determine $\varepsilon_{x,}\ \varepsilon_y\ and\ \varepsilon_{xy}$ at the point x = 1, y = 0.

Solution: $\varepsilon_x = \frac{\partial u}{\partial x} = (-2x + 6y)10^{-4}$

Put the value of x and y in above equation

= -2$\times\ 10^{-4}$

$$\varepsilon_y = \frac{\partial v}{\partial y} = (6 + -2y)10^{-4}$$

Again put the value of y, we get

= 6$\times\ 10^{-4}$

$$\varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = (4y + 6x)10^{-4} + (3)10^{-4}$$

= 9$\times\ 10^{-4}$

STRESS-STRAIN RELATIONS

For linear elastic materials, the stress- strain relations come from the generalized Hooke's law. For isotropic materials, the two properties $are\ Young's\ modulus\ (or\ modulus\ of\ elasticity), E\ and\ Poisson's\ ratio,$ v. Considering an elemental cube inside the body, Hooke's law gives

$$\varepsilon_x = \frac{\sigma_x}{E} - v\frac{\sigma_y}{E} - v\frac{\sigma_z}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - v\frac{\sigma_x}{E} - v\frac{\sigma_z}{E}$$

$$\varepsilon_z = \frac{\sigma_z}{E} - v\frac{\sigma_x}{E} - v\frac{\sigma_y}{E} \qquad (a)$$

$$\sigma_{yz} = \frac{\tau_{yz}}{G}$$

$$\sigma_{xz} = \frac{\tau_{xz}}{G}$$

$$\sigma_{xy} = \frac{\tau_{xy}}{G} \qquad (b)$$

Where

G is shear modulus (or modulus of rigidity) and G is given by

$$G = \frac{E}{2\,(1+v)} \qquad (c)$$

From Hooke's law relationships,

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{(1-2v)}{E}\left(\sigma_x + \sigma_y + \sigma_z\right) \qquad (d)$$

Substituting for $(\sigma_y + \sigma_z)$ and so on into (d), the resulted inverse relations are:

$$\sigma = D\varepsilon \qquad \text{(e)}$$

Where D is the symmetric (6x6) material matrix given by

$$D = \frac{E}{(1+v)(1-2v)}\begin{bmatrix} 1-v & v & v & 0 & 0 & 0 \\ v & 1-v & v & 0 & 0 & 0 \\ v & v & 1-v & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5-v & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5-v & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5-v \end{bmatrix} \qquad \text{(f)}$$

<u>One Dimension</u>

In one dimension, the normal stress σ is along x direction and the corresponding strain is ε. The stress- strain relation is:

$$\sigma = E\varepsilon \qquad \text{(g)}$$

Two Dimensions

In two dimensions, the problems are modelled as plane stress and plane strain.

Plane Stress: It is a condition that prevails in a flat plate in the xy plane, loaded only in its own plane and without z direction restraint, so that $\sigma_z = \sigma_{yz} = \sigma_{zx} = 0$. Then, for isotropic and isothermal conditions

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} = \frac{E}{1-v^2} \begin{bmatrix} 1 & -v & 0 \\ v & 1 & 0 \\ 0 & 0 & \frac{1-v}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} \qquad \text{(h)}$$

Plane Strain: It is defined as a deformation state in which w = 0 everywhere and u and v are functions of x and y, but not of z thus, $\varepsilon_x = \varepsilon_{yz} = \varepsilon_{zx} = 0$.The state of plane strain occurs in members that are not free to expand in the direction perpendicular to the plane of the applied loads. A typical slice of an underground tunnel that lies along the z- axis might deform in essentially plane strain conditions. Then, for isotropic and isothermal conditions

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} = \frac{E}{(1+v)(1-2v)} \begin{bmatrix} 1-v & v & 0 \\ v & 1-v & 0 \\ 0 & 0 & \frac{1-2v}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} \qquad \text{(i)}$$

EXAMPLE 1 In a plane strain problem, we have $\sigma_x = 1400MPa, \sigma_y = -100\ MPa, E = 200GPa$ and V = 0.3. Determine the value of the stress σ_z.

Solution: For a plane strain problem, we get

$$\begin{Bmatrix} 1400 \times 10^6 \\ -100 \times 10^6 \\ \sigma_{xy} \end{Bmatrix} = \frac{200\times10^9}{(1+0.3)(1-0.6)} \begin{bmatrix} 1-0.3 & 0.3 & 0 \\ 0.3 & 1-0.3 & 0 \\ 0 & 0 & \frac{1}{2}-0.3 \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix}$$
(1)

Therefore, Eq. (1) results

ε_x = 6.565$\times\, 10^{-3}$ and $\varepsilon_y = -3.185 \times 10^{-3}$

From Hooke's law relationships,

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{(1-2v)}{E}(\sigma_x + \sigma_y + \sigma_z)$$

And also for a plane strain problem, we get

$$\varepsilon_z = 0$$

$$6.565\times 10^{-3} - 3.815 \times 10^{-3} + 0.0 = \frac{1-0.6}{200\times10^9}(1400 \times 10^6 - 100 \times 10^6 + \sigma_z \qquad (2)$$

The solution of Eq. (2) results

$$\sigma_z = 390MPa$$

In the plane strain conditions, even though the strain, $\varepsilon_z = 0$; the stress,σ_z need not be zero.

Thank You

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