Computers & Structures Vol. 12, pp. 361–370
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0045-7949/80/1001-0361/$02.00/0

THE FINITE ELEMENT METHOD AFTER TWENTY-FIVE YEARS: A PERSONAL VIEW

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(*Received* 20 *September* 1979; *received for publication* 24 *January* 1980)

Abstract—The purpose of this paper is to examine the current state of development of the finite element method with regard to engineering applications. First is presented a personal view of the origins of the method, describing the sequence of events at Berkeley. Next is a discussion of the state-of-the-art of structural dynamic analysis, with mention of important recent advances. Finally, two examples drawn from earthquake engineering experience are discussed which demonstrate some limitations of present capabilities. Specific areas requiring new program development are mentioned; the need for a combined analytical-experimental approach is emphasized.

1. INTRODUCTION

It is now over twenty-five years since the finite element method first was used in the solution of practical structural engineering problems. Therefore, it may be appropriate at this time to examine the accomplishments of this past quarter century of phenomenal development. This task has been undertaken with trepidation because the subject has grown in breadth and refinement to the point where it cannot be evaluated adequately by any one observer. Nevertheless, it is hoped that this highly subjective view may be of some value because it is a product of a continuing interest in and contact with the finite element method during this entire period.

Admittedly, taking a backward view like this suggests that one is no longer looking ahead, and this implication is at least partially valid in the present case because the direction of my research has changed. Almost a decade age I became concerned that the advancement of structural analysis capabilities was progressing much more rapidly than was knowledge of the basic material and structural component behavior mechanisms, at least for the nonlinear response range. This deficiency of experimental data was particularly evident in the field of earthquake resistant design, where the structural performance must be evaluated during large cyclic excursions into the nonlinear range. Therefore, during most of the past decade I have followed this alternate path of dynamic experimental research, and have been involved only peripherally with recent developments in the finite element field.

At the outset of this review, it is important to express my concern over the tendency for users of the finite element method to become increasingly impressed by the sheer power of the computer procedure, and decreasingly concerned with relating the computer output to the expected behavior of the real structure under investigation. This concern is similar to that expressed last August by Oden and Bathe in their "Commentary on Computational Mechanics"[1], wherein they decried the complacency and overconfidence of "number-crunching" experts. They illustrated this attitude by the opinion of one such expert that "within the next decade the only use aerodynamicists will have for wind tunnels is as a place to store computer output". To the technologist who communicates only with a computer this may seem like a reasonable assessment; but the opinion certainly is not shared, for example, by engineers designing for wind loading on buildings who can estimate such loads only from boundary layer wind tunnel experiments. In fact, experimental evidence on how structures actually behave is usually the surest cure for overconfidence on the part of computer enthusiasts, and I am pleased to note that there is a growing trend toward experimental verification of the results of extensive computer calculations.

In order to provide a broad commentary on the development of the finite element method, this paper has been divided into three parts. First is a look back to the early days of the finite element development, when the method was viewed in the context of an extension of standard methods of structural analysis. This emphasis on application to the solution of real problems of engineering practice is reflected in the remainder of the paper as well. The second part is an assessment of "recent" advances in structural dynamic analysis capabilities, where the point of reference is a pair of "state-of-the-art" papers written 10 and 7 years ago. Emphasis is placed on dynamic analysis because the static response may be looked upon as a special case of the dynamic problem. In the third part of the paper, difficulties encountered in the analysis of two types of structures will be discussed. The purpose of this section is to demonstrate that major problems still remain to challenge the ingenuity of the finite element researcher, and also that experimental research is essential to discovering, defining, and eventually solving many of these problems. In this latter opinion, I tend to differ from the viewpoint expressed in the paper by Oden and Bathe[1] which emphasized the necessity for fundamental research in applied mechanics and mathematics as the primary approach to a wide range of unsolved problems in computational mechanics.

2. EARLY DEVELOPMENTS

As with any major development in engineering or mechanics, the early history of the finite element method (FEM) can be traced along several paths; certainly no single view of the origins can cover all facets of the development. Moreover, as more individuals and organizations began working with this engineering tool, the advances become increasingly diffuse—so detailing the history for more than a few years quickly becomes a task for specialists in the history of engineering science. Accordingly, this account does not pretend to provide a definitive history of the FEM; instead it merely gives some personal observations on the early days as seen by one of the participants.

It was the words "Engineering Application" in the name of this Conference that encouraged me to prepare this historic summary because the early development of the FEM with which I was involved was continually directed toward engineering applications. Initially, it was a tool designed and developed to solve real engineering problems, even though that outlook is quite distinct from much of the research presently being done in the finite element field.

It isn't possible to identify the exact starting point of the finite element method; and to spend time in conjecture on this question is a meaningless exercise, because the method makes use of many theories and techniques drawn from mathematics and continuum mechanics. One aspect of the FEM, mathematical modeling of continua by discrete elements, can be related to work done independently in the 1930s by McHenry[2] and Hrennikoff[3]—formulating bar element assemblages to simulate plane stress systems. Indeed, I spent the summer of 1952 at the Boeing Airplane Company trying to adapt this procedure to the analysis of a delta airplane wing, the problem which eventually led to the FEM; but because the technique could not effectively deal with plates of arbitrary configurations this effort was soon abandoned.

A more significant preliminary to the development of the FEM was the matrix generalization of structural theory in which the analysis was formulated as a form of coordinate transformation. The earliest known references to the assembly of structural elements by a matrix coordinate transformation were by Falkenheimer[4] and Langefors[5]. However, the classic work which completely stated the matrix formulation of structural theory, and which clearly outlined the parallel procedures of the force and displacement methods, was the series of articles first published in Aircraft Engineering by Argyris *et al.*[6]. It was this work which demonstrated that the concepts of classical structural analysis can be generalized for application to assemblages of any type of structural elements, not only to the traditional beams, struts, etc.

However, the true finite element concept is concerned primarily with the discretization process, not with the procedure used to analyze the system after the discrete elements have been identified and evaluated. Specifically, the FEM discretization involves the assumption of strain or stress fields defined on a regional basis, rather than replacement of the actual continuum by a set of substitute elements. Of course, this general concept applies to well known approximation methods of continuum mechanics, such as the Rayleigh–Ritz method, and it is true that the FEM may be looked upon as a special form of such methods. But the unique feature of the FEM was the idea of defining the strain field independently for the various regions or elements into which the continuum was divided.

Although this regional discretization concept had been proposed earlier[7, 8], it was only when it was used by an engineering organization as a means of avoiding the difficulty of physical discretization by bar assemblages that the method really began to develop. In addition, the concurrent availability of effective digital computers and of the matrix formulation of structural analysis were essential factors in the early development. Also, to maintain proper perspective on the early days of the FEM, it is important to realize that its utilization did not take off explosively; during the first six or seven years the application of the method spread very slowly indeed.

The work which I associate with the beginning of the computerized FEM was done during summer 1953 when I was again employed by Boeing Airplane Company on their summer faculty program. Again, I was assigned to Mr. M. J. Turner's Structural Dynamics Unit, to work on methods of evaluating the stiffness of a delta airplane wing for use in flutter analysis. Because the bar assemblage approach tried during the previous summer had been unsatisfactory, Mr. Turner suggested that we should merely try dividing the wing skin into appropriate triangular segments. The stiffness properties of these segments were to be evaluated by assuming constant normal and shear stress states within the triangles and applying Castigliano's theorem; then the stiffness of the complete wing system (consisting of skin segments, spars, stringers, etc.) could by obtained by appropriate addition of the component stiffnesses (the direct stiffness method). Thus, at the beginning of the summer, 1953, Mr. Turner had completely outlined the FEM concept and those of us working on the project merely had to carry out the details and test the results by numerical experiment.

Our paper describing this initial effort was presented at the New York meeting of the Institute of Aeronautical Sciences in January 1954[9]. I have never known why the decision was made not to submit the paper for publication until 1955, so the publication date of September 1956 was more than two years after the first presentation and over three years after the work was done. As was mentioned, this is graphic evidence that the FEM did not attain instant recognition. Undoubtedly, a major factor which limited its acceptability was that the original work was done in the Structural Dynamics Unit, where the objective was limited to stiffness and deflection analysis; it was several years before the concept was accepted and put to use by the stress analysis groups at Boeing. Thus, it is possible that the orientation of this initial step toward a specific engineering application tended to obscure the general applicability of the FEM concept, even though the individuals working with the development at Boeing were quite aware of its broader implications.

Although I maintained close contact with several of my Boeing colleagues for many years after 1953, I did not work there again and I had no opportunity for further study of the FEM until 1956–57, when I spent my first sabbatical leave in Norway (with the Skipsteknisk Forsknings Institutt in Trondheim). This "Norwegian connection" also was a factor in my decision to prepare a historical summary for this Conference; it was this period which made possible my continued contact with the finite element concept. Lack of computer facilities in Norway limited the type of work I could do at this time, but I was intrigued by plane stress applications of the method and I carried out some very simple analyses of rectangular and triangular element assemblages using a desk calculator. Although this work was too trivial to warrant publication, it convinced me of the potential of the FEM for the solution of general continuum problems.

About the time I returned to Berkeley from my sabbatical leave, the Engineering College acquired an IBM 701 Computer (replacing the old Card Programmed Calculator) and we began to develop structural analysis capabilities with this machine. For educational purposes, a Matrix Interpretive Program of the type pioneered in England[10] offered the best means of making the computer capabilities accessible to the students, and most of my early efforts went into developing such a program[11]. Then it was possible to continue my work with the FEM, which had been undergoing continuing development at Boeing but had attracted only little attention elsewhere.

Early FEM studies at Berkeley were greatly limited by the two thousand word central processor capacity of the IBM 701, but by utilizing the seven 2000 word drum storage units it was possible to carry out some creditable analyses. Our first concentrated effort toward plane stress analysis was in response to a challenge by one of my continuum mechanics colleagues who was sceptical of the validity of the procedure and wanted to see a solution of some classical problem. To me it seemed obvious that the method could solve any plane stress problem to any desired accuracy—limited only by the time and energy one wished to expend on the calculations. But in the hopes of attracting wider interest toward the FEM concept, I allocated part of a small NSF research grant to the solution of a few sample plane stress problems. The results were as good as I had expected, so a paper was prepared. The principal problem that arose in writing the paper was choosing a suitable name for this analytical procedure and I decided finally on the Finite Element Method. This name first appeared in that paper[12], and I can only conclude from subsequent history that it was an apt choice.

In retrospect, the next red letter event in my personal FEM history occurred in December 1960, when Professor O. C. Zienkiewicz invited me to Northwestern University to give a seminar lecture on the new procedure. We were friends from previous meetings, and I knew that he had been brought up in the Southwell finite difference tradition, so it was apparent that his invitation was prompted by scepticism and a desire to discuss the relative merits of finite elements vs finite differences. Certainly, we did have such discussions during my visit, but Professor Zienkiewicz obviously is a very intelligent person and was quick to recognize the advantages of the FEM. During that short visit an illustrious convert was won to the cause, and I think it is not coincidental that rapid worldwide acceptance of the FEM started almost from that moment.

The 1960 paper was not a major work because it described no new ideas; it was useful mainly in introducing the FEM to the Civil Engineering profession. My principal interest at that time was in developing the method as a general purpose tool for the analysis of arbitrary shell structures. One of my students already had worked on the FEM analysis of plate bending, but when we were about to prepare this paper for publication I learned that my former associates at Boeing already had prepared an internal report on plate bending analysis using finite elements. So this report was merely filed with my sponsors at NSF[13] and my student continued with the next step of combining plate bending and plane stress stiffness to obtain flat plate shell elements. Rectangular elements were derived for analysis of cylindrical shells and triangular elements for shells of arbitrary geometry. Significant results were obtained[14], but even in this early work it was apparent that an *ad hoc* approach to extending the FEM left much to be desired. Specifically the triangular plate bending element was not performing satisfactorily and it was necessary to go back for another look at that problem. Results of a follow-on study[15] were still less than satisfactory and it was not until 1964 that an adequate solution had been obtained for this problem.

The other important FEM development at Berkeley in the early 1960s was that we obtained a research contract from the U.S. Corps of Engineers for a practical engineering investigation; specifically they were concerned with the state of stress in a concrete gravity dam that had developed a major internal crack during construction due to temperature effects[16]. This contract provided us for the first time with enough money to support the development of a general purpose plane stress analysis program[17]. It was this program that subsequently proved to many engineers the great power of the FEM; even more important, writing the program allowed my student at that time, E. L. Wilson, to develop and demonstrate his great flair for finite element work. We at Berkeley are greatly indebted to the Corps of Engineers for their contribution to this cause, as well as for support of our finite element research through many years.

One other aspect of the early development of the FEM should be mentioned—the important role played by international conferences. To a large extent, the rapid worldwide expansion of interest and activities in the method is attributable to these conferences, which allowed for formal presentations of ideas and for personal meetings between the active researchers. To my knowledge, the first truly international conference dealing extensively with computer analysis of structures was held in Lisbon in September 1962[18]. The subject of this conference was Numerical Methods in Civil Engineering, and the FEM was the central theme of only one paper; but the conference provided a good forum for discussion of the relative merits of the finite difference and finite element procedures in structural analysis. By 1965, when the first Conference on Matrix Methods in Structural Mechanics was held at the Wright Patterson Air Force Base[19], the expansion of interest in and activity with the FEM was phenomenal. This milestone event brought together from all over the world nearly all researchers who had done significant work with finite elements. At the conclusion of the Conference, it was evident that the FEM had come of age; its potential for solving practical problems had been demonstrated in many structural disciplines, and powerful computer programs had been described which could deal routinely with problems of every description. Of course tremendous advances in understanding and in computational capability have been made since 1965, but this Conference, held only about a decade after the first preliminary applications, showed that the FEM should be recognized as the major analytical tool in the field of structural mechanics.

3. RECENT ADVANCES

Advances in finite element methodology since 1965 have been so rapid and diverse that it is impossible to chronicle them here, even considering only a very limited segment of this rapidly broadening field of mechanics. However, it may be useful to continue this personal view with a very limited discussion of the directions in which more recent work has been headed. The point of departure for this review is the 1969–72 state-of-the-art of structural dynamic response analysis, as summarized in two papers prepared for the U.S.–Japan Seminars on Matrix Methods of Structural Analysis[20, 21]. As was mentioned earlier, this is about the time that my interest turned toward experimental research; so it is evident that my comments on these recent developments present the views of an interested observer, but not an active participant in finite element research.

The field of dynamic response analysis is appropriate for the present discussion if the static case is considered merely as a special case of the dynamic problem, in which inertial and damping effects are not involved, and where equilibrium need be satisfied only at one time rather than at a succession of times during the response. In the 1969 and 1972 papers, the response analysis procedure was divided into two phases: (a) formulation of the equations of dynamic equilibrium, and (b) solution of these equations in response to the given condition of dynamic loading. It will be convenient to discuss these two phases separately here.

3.1 *Formulation of the equations of motion*

Establishing the equations of equilibrium in a finite element analysis involves three essential steps: (1) idealizing the actual structural system as an assemblage of discrete elements, (2) evaluating the mechanical properties of the elements, and (3) assembling the element properties to obtain the corresponding system properties. By 1969, the concept of finite element discretization was well developed, a wide range of element types was available for providing reasonable idealizations of arbitrary structures, and efficient coordinate systems and interpolation functions had been established for evaluating the element mechanical properties.

Moreover, procedures of assembling the element properties to obtain the system properties were well understood in 1969. To a great extent, these procedures are more closely associated with computer program coding than with finite element theory, and little need be said about them here. The only exception is the concept of substructuring, or more specifically the recognition that substructures can be looked upon as "superelements" and thus may be incorporated directly into the finite element assembly sequence. The substructure idea was well understood in 1969, but great progress has been made since then in designing programs that use multilevel substructuring as a routine feature of the analysis procedure[22].

Thus the only phase of formulation of the equations of motion remaining to be mentioned is the evaluation of the element properties, which for a dynamic analysis include the stiffness, mass, damping and external load. In this discussion each of these properties will be treated in turn.

3.1.1 *Stiffness*. By far the most significant mechanical property in most structural problems is the stiffness; accordingly the majority of finite element research has been devoted to developing more efficient elements which provide a better approximation of the actual structure stiffness at less computational cost. By 1969 great progress had been made in this direction, and although much research has gone into this area since then, I think it has been with considerably diminishing returns. Although some significant improvements of element types have been made, these have had little influence on most practical engineering applications. A recent paper by Olsen[23] supports this opinion; he noted that the old triangular flat plate element still remains competitive in analysis of general thin shells, in spite of extensive work done with refined shell elements.

3.1.2 *Mass*. An effective description of the inertial resistance of a structure is fundamental to a dynamic response analysis, and the consistent mass concept[24] was introduced early in the history of finite elements to provide a rational basis for forming mass matrices. However, a strong case was made in the 1969 paper for lumped mass representation rather than consistent mass, on the basis that inertial effects required a less refined discretization than did the elastic resistance. Subsequent research has supported this opinion, and some recent research has been concerned with developing procedures for effectively lumping the mass of arbitrary elements[25].

3.1.3 *Damping*. In contrast to the other mechanical properties, definition of damping or energy loss characteristics continues to be an elusive problem. Because so little is known about the actual damping processes, the damping property generally is defined at the assemblage level rather than for individual elements. In most cases a viscous damping mechanism seems to give adequate correlation between observed structural behavior and analytical predictions; experimental data usually is too limited to warrant development of more refined mathematical formulations.

Effective procedures were described in the 1969 paper and subsequently[26] for explicit definition of a proportional viscous damping matrix. Considerable interest has arisen since then in non-proportional damping, due in part to concern over "radiation damping" effects associated with soil-structure interaction. The classical techniques of treating non-proportional damping (based on mode-superposition using the complex mode shapes) have been employed in some finite element analyses[27]; but a preferable approach in most earthquake engineering problems is to transform to a reduced set of undamped modal coordinates, and then to solve the coupled modal equations[28]. The cost of solving the complex eigenproblem associated with non-proportional damping, and the subsequent cost of expressing the response in complex modal coordinates far exceeds the cost of calculating the response with a coupled set of coordinates.

Another major recent development in treating damping, which will be mentioned later, is through the use of a frequency domain solution. But the major fact which continues to limit the treatment of damping in dynamic systems is the paucity of experimental data concerning the actual energy loss mechanisms in real structural systems.

3.1.4 *Load*. Definition of the load terms in the equations of motion also generally is done at the level of the assembled structure rather than with respect to the elements, and the principal problems with this factor usually are due to lack of knowledge of the true loading mechanisms rather than with the discretization process. Therefore, progress in this area depends more on obtaining experimental data than on finite element research, and little change can be reported since 1969. Significant advances have been made recently in related areas involving fluid-structure interaction and foundation-structure interaction, especially in response to earthquake excitation; these topics will be mentioned briefly later, but generally they are outside the scope of the present discussion.

3.2 *Solution of the equilibrium equations*

In discussing the second phase of the finite element analysis, the solution of the equations of equilibrium, it is useful to divide the subject into several types of categories. First, static and dynamic problems will be separated because of the radically different analytical procedures that may be employed in the solution of the dynamic problem. Then the dynamic analyses may be separated according to the type of coordinates used in the solution: modal or discrete system coordinates. Another classification concerns the "domain" of the analysis procedure: time domain or frequency domain.

An alternative approach to categorization would be to separate linear problems from nonlinear. In discussing the nonlinear analyses, a generally similar approach could be adopted for both static and dynamic cases, the latter merely requiring the inclusion of extra terms to represent the effects of inertia and damping. However, the first approach is more satisfactory for the purpose of this paper.

3.2.1 *Static analysis.* Because the development of efficient algorithms for the solution of large equation sets has been a primary objective for finite element researchers since the method first was put to practical use, the capability in this area already was quite advanced by 1969. Powerful equation solvers were available then, and extensive research since 1969 has further advanced the state-of-the-art. Undoubtedly many of the most significant recent advances have pertained to analysis on nonlinear structures because the nonlinear analysis cost for large practical systems has been almost prohibitive. The continuing active interest in this subject is evidenced by the many major international conferences recently devoted to it[29, 30]; also it is significant that two sessions of this present conference deal with nonlinear analysis.

3.2.2 *Dynamic coordinates.* The first major decision to be made in planning a dynamic response analysis is the type of coordinates to be used. One option is merely to solve directly the equations of motion expressed in the original system coordinates, but this has the disadvantage that the original equation set may involve hundreds or thousands of degrees of freedom. An alternative is to transform to the natural or modal coordinates of the structure which describe the dynamic response more efficiently, and therefore, may be reduced in number. Each of these approaches will be discussed briefly.

(a) *Modal coordinates*

If the structure is linearly elastic, it is usually desirable to transform the equations of motion to modal coordinates and then to evaluate the response in terms of a truncated modal set. Solution of the structure eigenproblem to obtain the modal coordinates is a major computational task, and very effective eigenproblem solvers were developed quite early, as is described on the 1969 and 1972 papers. Research in this area has continued since then, and further refinements of technique are being made, but it is not likely that a major breakthrough will result.

The principal decision facing the analyst in a modal coordinate solution is the number of modes to be included. Clearly this depends on both the spatial distribution and the frequency content of the applied loading. If the load pattern is distributed widely over the structure, it tends to excite mainly the lower modes of vibration which may have distribution patterns similar to the loading. However, if only a small portion of the structure is attacked by the external load, many modes will be excited.

In the past it generally has been assumed that all modes which are excited significantly must be included in the dynamic response analysis. However, it is evident that the amplitude of the dynamic response in any mode depends on the frequency content of the loading. The higher mode response to a low frequency excitation may be essentially static in character, and recently[31] it has been demonstrated that these modal contributions can be accounted for adequately by a static correction term. Thus a standard mode superposition response is performed for only those modes which are subject to dynamic amplification; the response to the load associated with the higher modes is evaluated by a static analysis, and this static correction is added to the dynamic response.

In general, the modal coordinate approach is not recommended for nonlinear analysis; however, some studies have shown that efficient nonlinear solutions can be obtained with modal coordinates[32, 33]. Clearly this approach can be effective only if the dynamic response can be expressed conveniently by superposition of the mode shapes, and therefore it is applicable to systems in which the nonlinearity does not drastically modify the vibration shapes. In principle, one would expect good performance where the nonlinearity is widely distributed over the structure, but not where local concentrations such as plastic hinges are involved.

(b) *System coordinates*

Integration of the equations of motion expressed in system coordinates is the standard approach to nonlinear analysis, but significant savings may be effected if the structure has only localized nonlinearity[34]. In this case a substructure analysis procedure may be adopted wherein all nonlinear parts of the structure are included in a single substructure. The elastic portions of the system are included in other substructures, and the degrees of freedom not required for interconnection to the nonlinear component are removed by condensation. Thus the number of degrees of freedom required for consideration in the nonlinear analysis may be greatly reduced from the original coordinate set.

3.2.3 *Dynamic domain.* The other basic option open to the dynamic analyst is the domain in which the analysis will be performed. In 1969–72 this question seldom arose—step-by-step integration of the equations of motion was the only technique capable of dealing with large scale dynamic applications of the FEM. During the past decade, however, the extremely efficient Fast Fourier Transform (FFT) programs for numerical evaluation of Fourier transforms have been discovered by structural engineers, and have proven to be advantageous in many practical cases. Comments on both frequency domain and time domain analyses follow.

(a) *Frequency domain*

Frequency domain analyses involve the superposition of the response of a structure to various harmonic excitations; therefore, they are only applicable to linear systems. Moreover, because the analysis must be performed for many hundreds of frequencies it will be extremely expensive when dealing with systems having many coupled degrees of freedom. Therefore, frequency domain calculations generally are performed upon uncoupled modal coordinate sets.

One of the principal areas of application of the frequency domain approach to dynamic structural analysis is in situations involving liquid-structure or soil-structure interaction, because the continuous medium can be treated without discretization and coupled directly to the finite element model of the structure. Many important applications of this type of analysis have been reported in recent years[35, 36], and this is one of the significant areas of advances since 1969. Another practical application of frequency domain analysis is deconvolution, i.e. evaluating the input loading history from measured response data[37]. Such analyses can be performed easily in the frequency domain but are very difficult to do satisfactorily in the time domain. Except for special types of problems such as those mentioned above, however, frequency domain analyses seldom are superior to direct time domain integration of the equations of motion.

(b) *Time domain*

Improvement in understanding of the process of direct integration of the equations of motion is one of the most significant achievements of structural dynamics research during the past decade. The step-by-step analysis procedures described in the 1969 and 1972 papers are still valid and widely used, but research done subsequently[38–40] shows that these methods are merely members of much broader families of analysis techniques. These more recent studies (and the references cited above are merely arbitrary examples of numerous papers in this field) show that it is possible to design a step-by-step procedure to have desired approximation characteristics with respect to such factors as period elongation and artificial damping. However, the older methods, even going back 20 years to the pioneering work by Newmark[41] still serve well in many current applications.

4. EXAMPLES OF CURRENT PROBLEMS

Although remarkable progress has been made with the FEM during its first quarter century, and the basic procedures of structural dynamic analysis have advanced to the point where solution of arbitrary dynamic problems appears to be routine, it is obvious to those working with general engineering applications of the method that new problems and difficulties are encountered regularly. In this section of the paper two examples of problems that have confronted the author during the past year are described. These problems are viewed from the standpoint of an FEM user rather than a program developer; they illustrate limitations of analysis capabilities presently available to the user, and therefore, are offered as challenges to those involved in program development.

The two examples are drawn from earthquake engineering and concern the seismic response of (a) thin shell metal cylindrical liquid storage tanks, and (b) thin shell concrete arch dams. Although both involve liquid-structure interaction, the problems identified in the two cases are fundamentally different. However, both demonstrate that "number-cruncher complacency" is premature at this stage of development of the FEM, and that experimental research is essential to further progress—both in understanding of mechanical processes and in verification of analysis procedures.

4.1 *Response of liquid storage tanks*

Examples of damage to ground supported liquid storage tanks are noted after nearly every major earthquake, and designer efforts to improve their earthquake resistance have been under way for over twenty years. The present design procedure is based on an approximate evaluation of the hydrodynamic pressures induced by the earthquake, assuming that the tank is rigid. From these pressures the base shear and overturning moment are determined, and then the resulting shell stresses are calculated by elementary beam theory, treating the tank as a vertical cantilever.

An obvious limitation of this approach is the rigid tank assumption; a thin metal tank can be expected to deform sufficiently during the seismic response to permit significant dynamic amplification of the liquid pressures. Accordingly, several investigators have developed finite element programs[42, 43] which are intended to account for this seismic fluid-structure interaction. The basic assumption of these programs, drawn from hydrodynamic theory, is that a horizontal base motion along one axis will produce only a first Fourier harmonic ($\cos\theta$) distribution of pressure and deformation. Thus a single term axisymmetric shell analysis procedure has been adopted.

In order to verify results of such analyses (which are applicable only to fixed base tanks) and also to provide quantative information on the response of unanchored tanks, a group of tank manufacturers and designers sponsored an experimental study at the University of California using the 20×20 ft shaking table facility[44]. Figure 1 shows a "tall" tank on the shaking table, surrounded by (but not in contact with) its reference frame. It may be noted here that unanchored tanks are used frequently because they do not require the expensive foundations needed for anchorage, but even moderate earthquake motions will induce rocking uplift of the tank walls from the foundation and only very crude estimates of this dynamic response mechanism are presently available to designers.

Probably the most significant and surprising result of this test program was the observation that fixed base tanks did not respond only in the first Fourier harmonic; important $\cos 2\theta$ and $\cos 3\theta$ components of pressure, displacement and stress also were observed in tests of both short[44] and tall[45] tanks. Some of the tall tank results are included here to illustrate this unexpected response behavior. Figure 2 shows deflected shapes of the tank cross section at several times during a typical test, and Fig. 3 depicts the relative maximum amplitudes of the Fourier components contained in various response quantities. These figures clearly show that the standard finite element analysis, which recognizes only the $\cos\theta$ component, cannot provide much understanding of the actual fixed-base tank response.

In the case of the free-base tank, out-of-round response was expected because the axial symmetry of the base support condition is destroyed by uplift. However, comparison of the axial stress distribution about the base of the unanchored tank with an estimate based on a current design procedure, shown in Fig. 3, demonstrates the need for complete revision of the present design concept. Clearly the rocking mechanism concentrates the base section axial stress in a much narrower arc than had been expected, with the result that the measured stress exceeds the design estimate by a factor of about 4. The behavior in this case is highly nonlinear with regard to the support forces about the base, but no effort yet has been directed toward developing a nonlinear analysis procedure to deal with this important problem.

In the context of this paper, the first conclusion drawn from this example is that experimental verification is essential for ensuring reliability of finite element programs. The test of the anchored tanks had been looked

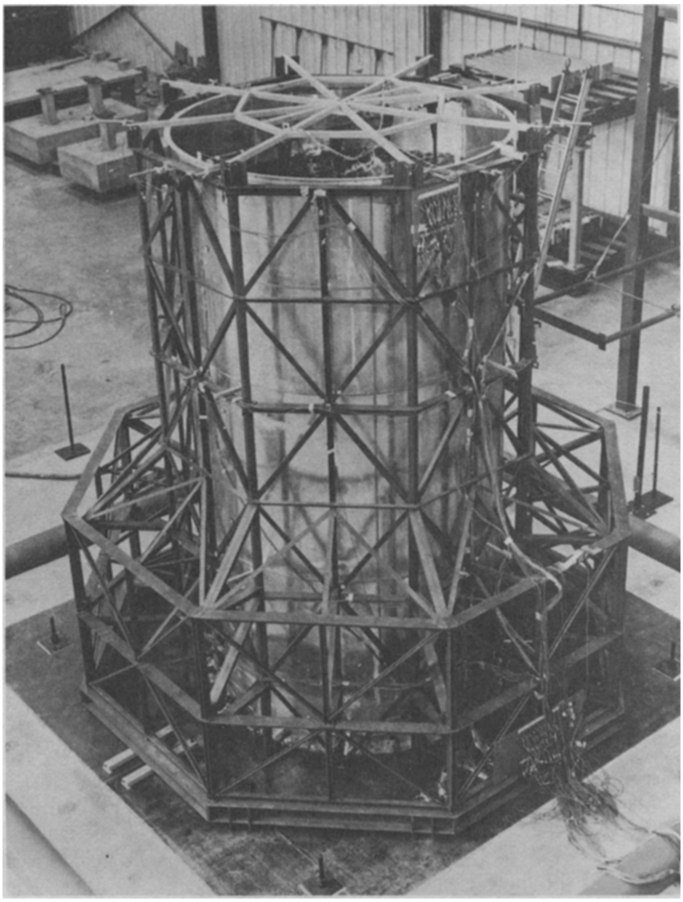

Fig. 1. 7 – 3/4 × 15 ft tank with reference frame on shaking table.

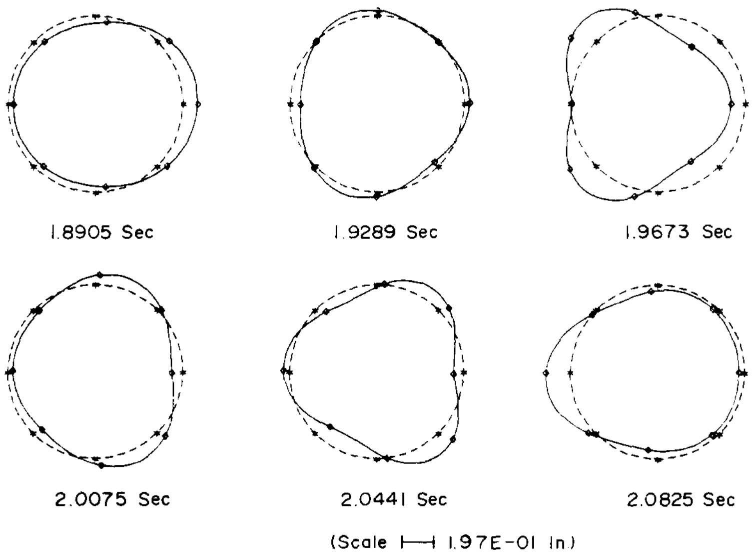

Fig. 2. Deflected shape of tank top during shaking table test.

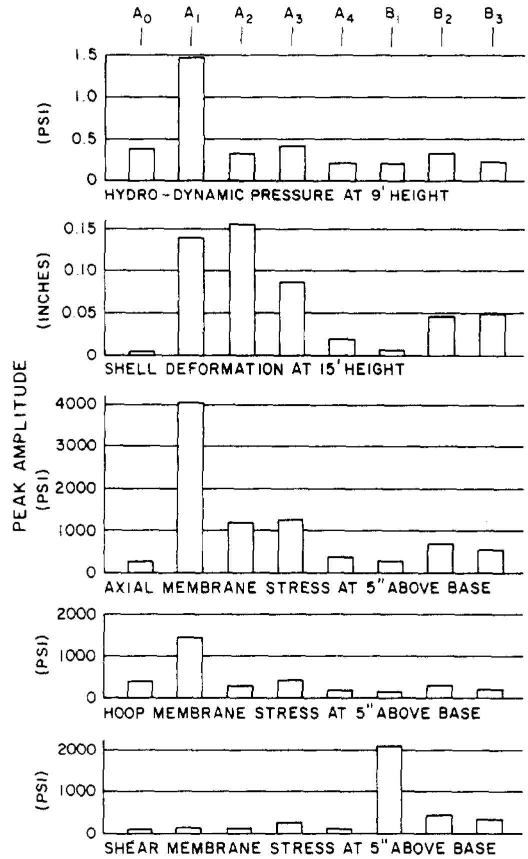

Fig. 3. Peak amplitude of Fourier components shaking table test of tank.

upon only as a routine check of the finite element approximation; the observed out-of-round response was totally unexpected. Theoretical mechanics has not yet provided a basis for predicting these response components, and of course, the finite element analysis cannot go beyond its theoretical origins no matter how refined the mesh or capable the computer. A second closely related conclusion is that experimental data is essential for developing adequate numerical approximations when theory is lacking. Certainly, future improvements in analysis techniques for either anchored or unanchored tanks will depend heavily on experimental results such as those mentioned here, and development of a finite element program which can deal effectively with either of these design problems is a major challenge to the profession of computational mechanics.

4.2 *Concrete arch dams*

The seismic performance of concrete arch dams is a problem of even more critical concern to engineers and the general public because a large reservoir could constitute a major hazard to the people living below it. Although the past earthquake performance of arch dams has been excellent, non-earthquake failures such as those at Malpasset, France and Vaiont, Italy demonstrate that dams embody a significant potential for disaster. Therefore, great emphasis presently is being placed on seismic safety evaluations of major dams in many regions having active seismicity.

Superficially the analysis of earthquake stresses in a concrete arch dam would appear to be well within the present state-of-the-art of dynamic finite element analysis. Indeed, computer programs have been written for that specific purpose[46], and general purpose programs such as SAPIV also have been used extensively in such studies. However, when these analyses are examined in detail their serious limitations become apparent. For example, reservoir interaction is treated only approximately at best—using "added masses" derived by means of incompressible liquid elements. However, two-dimensional gravity dam analyses have demonstrated the importance of liquid compressibility in the earthquake analysis of such structures[44], and it should be equally important in a three-dimensional arch dam analysis; yet no program accounting for arch dam-compressible liquid interaction is presently available.

Possibly even more critical in this arch dam-reservoir interaction behavior is the fact that added mass analyses indicate negative net fluid pressures in response to a severe earthquake. In other words, the peak negative dynamic pressures momentarily exceed the hydrostatic pressures, and therefore cavitation should result. However, no analysis procedure is available to deal with this nonlinear interaction mechanism, either for compressible or incompressible liquids.

The other major limitation of present seismic response studies for concrete arch dams is that they all treat the structure as a linearly elastic system. For the static design conditions this assumption is reasonable, but the response to a major earthquake is expected to include two significant forms of structural nonlinearity. The first results from the fact that typical arch dams are con-

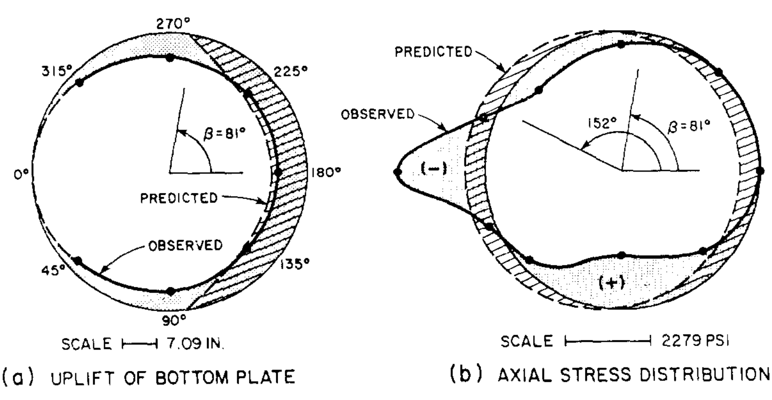

Fig. 4. Comparison of observed and predicted performance during test of unanchored tank.

structed as a series of independent monoliths separated by joints extending from foundation to crest and from upstream to downstream face. These joints are intended to minimize cracking due to shrinkage and temperature change during construction; after construction they are closed by grouting and/or the reservoir pressure on the upstream face, and no movement of the joints occurs during normal static load conditions. But during a severe earthquake, tension and bending effects are induced in the arch rings which can overcome the static compression and cause joint opening. Consideration of this joint behavior is essential to understanding of the seismic response; the tensile arch stresses indicated by linear dynamic analyses obviously are inconsistent with the actual jointed construction of the dam. Inclusion of this type of nonlinearity is well within present finite element analysis capabilities, but it has not yet been applied in any significant practical investigation.

The second type of structural nonlinearity expected during a major earthquake involves cracking of the concrete. Because the tensile strength of concrete is only about 10 percent of its compressive strength, stresses indicated by linear response analyses for a major earthquake often exceed the cracking limit. As was mentioned above, the vertical construction joints would prevent development of arch tensile stresses, but cracking would be expected on horizontal planes due to "cantilever" tensile stresses in the monoliths. Thus, during a severe earthquake an extensive network of horizontal cracks might develop, which could combine with the vertical joints to divide the dam into a system of separate blocks. Because the cracking tendencies are associated with high frequency vibrations, little relative displacement would be expected between the blocks; thus one would intuitively expect that the arch mechanism of the dam would not be adversely affected by such damage and that it would continue to support the gravity and water loads after the seismic input ended. However, such intuitive estimates seldom are convincing to Boards responsible for public safety; analytical procedures which can follow the history of cracking, joint opening, and block displacement, and can then evaluate the stability of the system, are urgently needed.

In concluding this discussion, it may be noted that the three types of nonlinearity identified in the seismic response of an arch dam—cavitation, joint opening, and cantilever cracking—each present formidable obstacles to a finite element analysis, and experimental data on the actual behavior would be required before any such program could be accepted for routine use. On the other hand, the difficulties of conducting a valid experimental study of these mechanisms also are great, because dynamic similitude requirements cannot easily be satisfied with the very small scale models that could be employed. Consequently, a combined analytical-experimental approach probably offers the best prospects for dealing with this important engineering problem.

5. CONCLUDING REMARKS

The purpose of this paper has been to provide some perspective on the background of the FEM and on its current capabilities in application to engineering problems. It is well to recall that the emphasis during early development of the method was oriented totally toward practical application. At present it probably is fair to say that the state-of-the-art has advanced to the point where solution of any structural engineering problem can be contemplated, but there may be a wide variation in the quality of the result obtained. Depending on the validity of the assumptions made in reducing the physical problem to a numerical algorithm, the computer output may provide a detailed picture of the true physical behavior or it may not even remotely resemble it. A controlling influence on where the final result lies along this scale is the skill of the engineer who prepares the mathematical idealization; when dealing with complex and unusual structures, this phase of the analysis is an art and the program cannot be treated merely as a "black box".

Because of the significant possibility that the analysis may have totally overlooked or misjudged some important aspects of the mechanical behaviour, experimental verification should be incorporated into the analytical process whenever it steps beyond the borders of experience and established practice. In addition, experimental studies often will be required when the analysis is breaking new ground in order to define and quantify the parameters which characterize the system. For these reasons, investigations of new structural systems should be planned by means of a combined analytical-experimental approach.

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