

## Chapter 03.04

### Newton-Raphson Method of Solving a Nonlinear Equation – More Examples

#### Civil Engineering

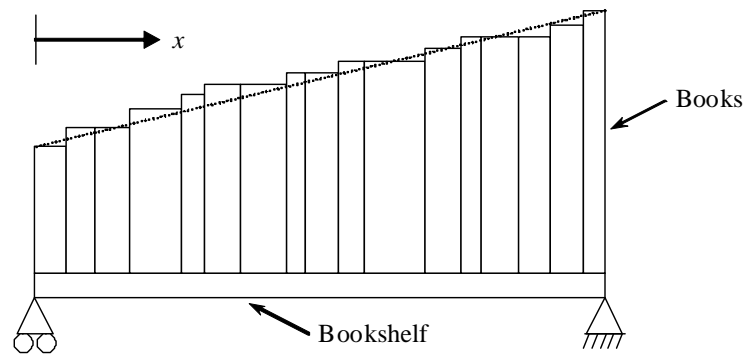
##### Example 1

You are making a bookshelf to carry books that range from 8½" to 11" in height and would take up 29" of space along the length. The material is wood having a Young's Modulus of 3.667Msi, thickness of 3/8" and a width of 12". You want to find the maximum vertical deflection of the bookshelf. The vertical deflection of the shelf is given by

$$v(x) = 0.42493 \times 10^{-4} x^3 - 0.13533 \times 10^{-8} x^5 - 0.66722 \times 10^{-6} x^4 - 0.018507x$$

where  $x$  is the position along the length of the beam. Hence to find the maximum deflection

we need to find where  $f(x) = \frac{dv}{dx} = 0$  and conduct the second derivative test.



**Figure 1** A loaded bookshelf.

The equation that gives the position  $x$  where the deflection is maximum is given by

$$-0.67665 \times 10^{-8} x^4 - 0.26689 \times 10^{-5} x^3 + 0.12748 \times 10^{-3} x^2 - 0.018507 = 0$$

Use the Newton-Raphson method of finding roots of equations to find the position  $x$  where the deflection is maximum. Conduct three iterations to estimate the root of the above equation. Find the absolute relative approximate error at the end of each iteration, and the number of significant digits at least correct at the end of each iteration.

**Solution**

$$f(x) = -0.67665 \times 10^{-8} x^4 - 0.26689 \times 10^{-5} x^3 + 0.12748 \times 10^{-3} x^2 - 0.018507 = 0$$

$$f'(x) = -2.7066 \times 10^{-8} x^3 - 0.80067 \times 10^{-5} x^2 + 0.25496 \times 10^{-3} x = 0$$

Let us take the initial guess of the root of  $f(x) = 0$  as  $x_0 = 10$ .

Iteration 1

The estimate of the root is

$$\begin{aligned} x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} \\ &= 10 - \frac{\left( -0.67665 \times 10^{-8} (10)^4 - 0.26689 \times 10^{-5} (10)^3 \right.}{\left. -2.7066 \times 10^{-8} (10)^3 - 0.80067 \times 10^{-5} (10)^2 + 0.25496 \times 10^{-3} (10) \right)} \\ &= 10 - \frac{-8.4956 \times 10^{-3}}{1.7219 \times 10^{-3}} \\ &= 10 - (-4.9339) \\ &= 14.934 \end{aligned}$$

The absolute relative approximate error  $|\epsilon_a|$  at the end of Iteration 1 is

$$\begin{aligned} |\epsilon_a| &= \left| \frac{x_1 - x_0}{x_1} \right| \times 100 \\ &= \left| \frac{14.934 - 10}{14.934} \right| \times 100 \\ &= 33.038\% \end{aligned}$$

The number of significant digits at least correct is 0, as you need an absolute relative approximate error of less than 5% for one significant digit to be correct in your result.

Iteration 2

The estimate of the root is

$$\begin{aligned} x_2 &= x_1 - \frac{f(x_1)}{f'(x_1)} \\ &= 14.934 - \frac{\left( -0.67665 \times 10^{-8} (14.934)^4 - 0.26689 \times 10^{-5} (14.934)^3 \right.}{\left. -2.7066 \times 10^{-8} (14.934)^3 - 0.80067 \times 10^{-5} (14.934)^2 \right)} \\ &\quad \left. + 0.12748 \times 10^{-3} (14.934)^2 - 0.018507 \right)}{\left( -2.7066 \times 10^{-8} (14.934)^3 - 0.80067 \times 10^{-5} (14.934)^2 \right.} \\ &\quad \left. + 0.25496 \times 10^{-3} (14.934) \right)} \\ &= 14.934 - \frac{6.9829 \times 10^{-4}}{1.9317 \times 10^{-3}} \\ &= 14.934 - (0.36149) \\ &= 14.572 \end{aligned}$$

The absolute relative approximate error  $|\epsilon_a|$  at the end of Iteration 2 is

$$\begin{aligned}
 |\epsilon_a| &= \left| \frac{x_2 - x_1}{x_2} \right| \times 100 \\
 &= \left| \frac{14.572 - 14.934}{14.572} \right| \times 100 \\
 &= 2.4806\%
 \end{aligned}$$

The number of significant digits at least correct is 1, because the absolute relative approximate error is less than 5% .

### Iteration 3

The estimate of the root is

$$\begin{aligned}
 x_3 &= x_2 - \frac{f(x_2)}{f'(x_2)} \\
 &= 14.572 - \frac{\left( -0.67665 \times 10^{-8} (14.572)^4 - 0.26689 \times 10^{-5} (14.572)^3 \right)}{\left( +0.12748 \times 10^{-3} (14.572)^2 - 0.018507 \right)} \\
 &= 14.572 - \frac{\left( -2.7066 \times 10^{-8} (14.572)^3 - 0.80067 \times 10^{-5} (14.572)^2 \right)}{\left( +0.25496 \times 10^{-3} (14.572) \right)} \\
 &= 14.572 - \frac{-4.7078 \times 10^{-9}}{1.9314 \times 10^{-3}} \\
 &= 14.572 - \left( -2.4375 \times 10^{-6} \right) \\
 &= 14.572
 \end{aligned}$$

The absolute relative approximate error  $|\epsilon_a|$  at the end of Iteration 3 is

$$\begin{aligned}
 |\epsilon_a| &= \left| \frac{x_3 - x_2}{x_3} \right| \times 100 \\
 &= \left| \frac{14.572 - 14.572}{14.572} \right| \times 100 \\
 &= 1.6727 \times 10^{-5}
 \end{aligned}$$

Hence the number of significant digits at least correct is given by the largest value of  $m$  for which

$$\begin{aligned}
 |\epsilon_a| &\leq 0.5 \times 10^{2-m} \\
 1.6727 \times 10^{-5} &\leq 0.5 \times 10^{2-m} \\
 3.3454 \times 10^{-5} &\leq 10^{2-m} \\
 \log(3.3454 \times 10^{-5}) &\leq 2 - m \\
 m &\leq 2 - \log(3.3454 \times 10^{-5}) = 6.4756
 \end{aligned}$$

So

$$m = 6$$

The number of significant digits at least correct in the estimated root 14.572 is 6.

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**NONLINEAR EQUATIONS**

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Topic	Newton-Raphson Method-More Examples
Summary	Examples of Newton-Raphson Method
Major	Civil Engineering
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