Module 16

Earthquake Resistant Design of Structures

Lesson 40

Ductile Design and Detailing of Earthquake Resistant Structures

Instructional Objectives:

At the end of this lesson, the student should be able to:

- define and explain ductility factor with respect to displacement, curvature or rotation,
- state the advantages of ductility in the design of reinforced concrete members,
- derive expressions of ductility factor of singly and doubly-reinforced rectangular beams,
- mention the factors influencing ductility,
- give general specification of materials for the ductile design of reinforced concrete members,
- state the general guidelines in the design and detailing of structures having sufficient strength and ductility,
- identify the situation when special confining reinforcement is needed,
- to determine the ductility factor of singly and doubly-reinforced rectangular beams applying the expressions,
- to apply the knowledge in designing and detailing beams, columns and beam-column joints as per IS 13920:1993.

16.40.1 Introduction

As mentioned in sec. 16.39.10 of Lesson 39, it is uneconomical to design structures to withstand major earthquakes. However, the design should be done so that the structures have sufficient strength and ductility. This lesson explains the requirements and advantages of ductility in the design of reinforced concrete members which can be expressed with respect to displacement, curvature or rotation of the member. The expressions of ductility of singly and doubly-reinforced beams with respect to curvature are derived. The influencing parameters of the ductility are explained. Several aspects of design for ductility are explained mentioning detailing for ductility, as stipulated in IS 13920:1993, for flexural members and columns. Illustrative examples are solved to determine the ductility with respect to curvature of singly and doubly-reinforced beams. Moreover, numerical problems are solved to illustrate the design of beams, columns and beam-column joints.

16.40.2 Displacement Ductility

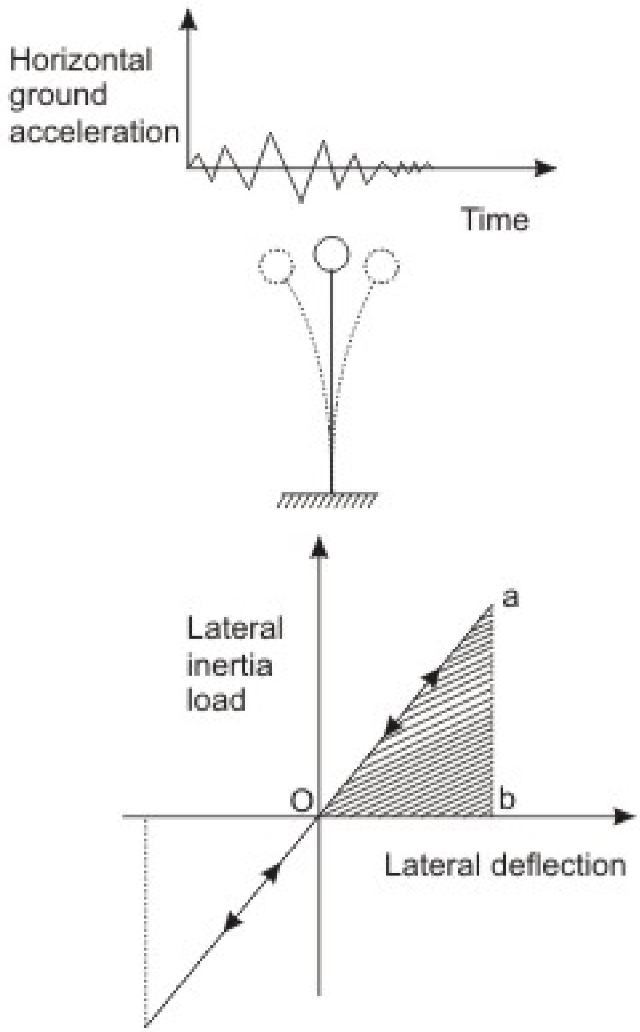


Fig. 16.40.1: Elastic response

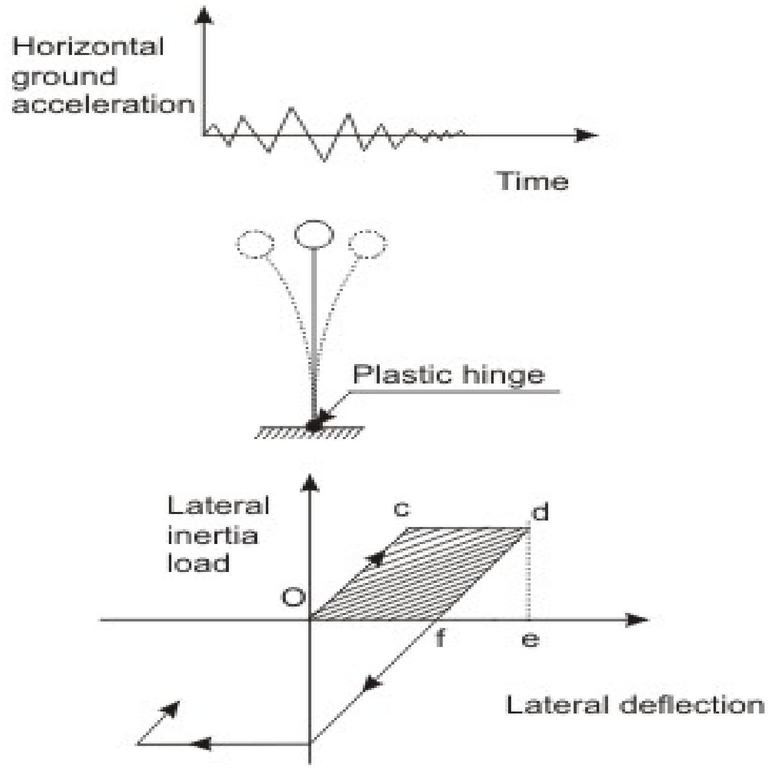


Fig. 16.40.2: Elasto-plastic response

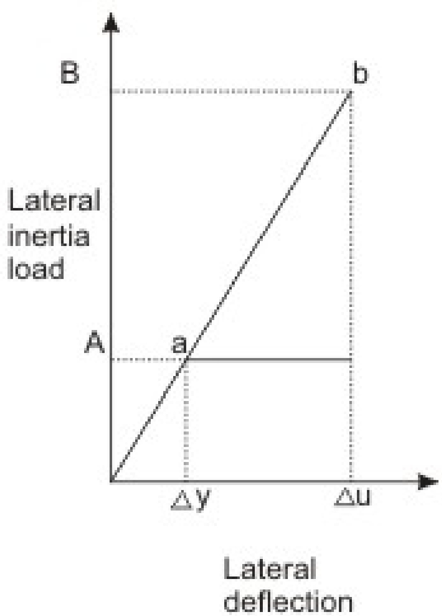


Fig. 16.40.3: Equal maximum deflection response

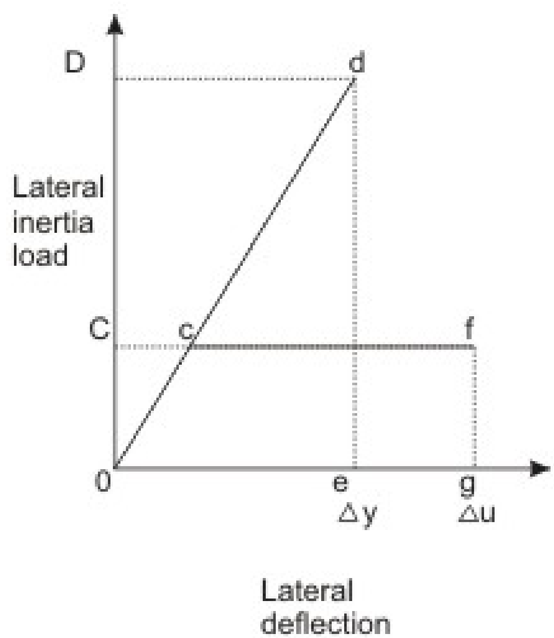


Fig. 16.40.4: Equal maximum potential energy response

It is essential that an earthquake resistant structure should be capable of deforming in a ductile manner when subjected to lateral loads in several cycles in the inelastic range. Let us take a single degree of freedom oscillator, as shown in Fig. 16.40.1. In the elastic response, the oscillator has the maximum response at a. The area oab represents the potential energy stored when maximum deflection occurs. The energy is converted into kinetic energy when the mass returns to zero position. Figure 16.40.2 shows the oscillator forming a plastic hinge at a much lower response c when the deflection response continues along cd, d being the maximum response. The potential energy at the maximum response is now represented by the area ocde. When the mass returns to zero position, the part of the potential energy converted to kinetic energy is represented by fde, while the other energy under the area ocdf is dissipated by the plastic hinge by being transferred into heat and other forms of energy, which are irrecoverable. It is thus evident that, elastically, the full potential energy is returned to kinetic energy, while elastoplastically a part of the energy is converted into kinetic energy. Hence, the potential energy stored in the elastoplastic structure may not be equal to that in elastic structure and the maximum deflection of the elastoplastic structure may not be equal to that of elastic structure. Figures 16.40.3 and 4 present equal maximum deflection and equal maximum potential energy responses of two structures.

The displacement ductility factor μ, a measure of ductility of a structure, is defined as the ratio of Δ_u, and Δ_y, where Δ_u, and Δ_y are the respective lateral deflections at the end of post elastic range and when the yield is first reached. Thus, we have

μ (with respect to displacement) = Δ_u / Δ_y (16.4)

The values of displacement ductility factor should range from 3 to 5.

16.40.3 Curvature Ductility

The curvature ductility factor μ is defined as the ratio of ϕ_u and ϕ_y, where ϕ_u and ϕ_y are the respective curvatures at the end of postelastic range and at the first yield point of tension steel, as stipulated in cl. 3.3 of IS 13920:1993. Thus, we have

μ (with respect to curvature) = ϕ_u / ϕ_y, (16.5)

It should be noted that the curvature ductility factor is significantly different from the displacement ductility factor. At the start of yielding in a frame, the deformations concentrate at the positions of plastic hinge. Therefore, when a frame is deflected laterally in the postelastic range, the ϕ_u / ϕ_y, ratio in a plastic hinge may be greater than $\Delta_u, / \Delta_y$ ratio.

16.40.4 Rotational Ductility

In a similar manner, the rotational ductility factor μ is defined as the ratio of θ_u and θ_y, where θ_u and θ_y are the respective rotations of at the end of postelastic range and at the first yield point of tension steel. Thus, we have

μ (with respect to rotation) = θ_u / θ_y
(16.6)

Thus, there are three methods of defining the ductility. In general, it can be stated that the ductility is the ratio of absolute maximum deformation at the end of postelastic range to the yield deformation. Accordingly, the ductility can be defined with respect to strain, rotation, curvature or deflection. Rotation and curvature based ductility factors take into account shape and size of the member. Though there is no special advantage of one or the other definition, we will consider curvature ductility, as stipulated in cl. 3.3 of IS 13920:1993 and explained in sec. 16.40.3 (Eq. 16.5).

16.40.5 Advantages of Ductility

The following are the advantages of a reinforced concrete structure having sufficient ductility:

(i) A ductile reinforced concrete structure may take care of overloading, load reversals, impact and secondary stresses due to differential settlement of foundation.

(ii) A ductile reinforced concrete structure gives the occupant sufficient time to vacate the structure by showing large deformation before its final collapse. Accordingly, the loss of life is minimised with the provision of sufficient ductility.

(iii) Properly designed ductile joints are capable of resisting forces and deformations at the yielding of steel reinforcement. Therefore, these sections can reach their respective moment capacities, which is one of the assumptions in the design of reinforced concrete structures by limit state method.

16.40.6 Expressions of Ductility of Reinforced Concrete Rectangular Beams

(A) Singly-reinforced rectangular section

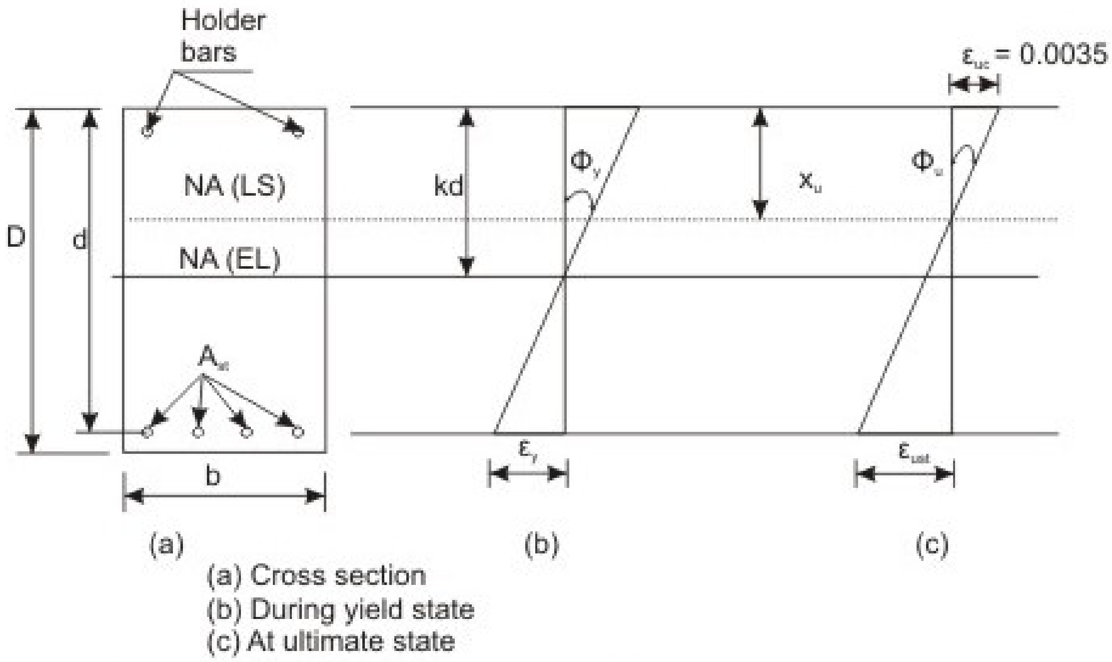


Fig. 16.40.5: Rectangular cross-section of a singly-reinforced beam

igures 16.40.5a, b and c show the cross-section of a singly-reinforced rectangular beam, strain profile and curvature at yield and ultimate stages. From Fig. 16.40.5b, we have:

$$\phi_y = \epsilon_y / (d - kd) \tag{16.7}$$

where $\epsilon_y = f_y/E_s$ (16.8)

$$k = -(mp/100) + \{(mp/100)^2 + 2\,(mp/100)\}^{1/2} \tag{16.9}$$

$$m = 280 / 3\sigma_{cbc}$$
(16.10)

and σ_{cbc} = permissible stress of concrete in bending compression.

From Fig. 16.40.5c, we have:

$$\phi_u = \epsilon_{uc} / x_u$$
(16.11)

where ϵ_{uc} = 0.0035, as given in cl. 38.1b of IS 456:2000.

From Eq. 3.17 of Lesson 5, we have for singly-reinforced sections,

$$x_u / d = 0.87\, f_y\, p \,/\, (36)\, (f_{ck})$$
(16.12)

where $p = 100\, A_{st} / bd$ (16.13)

It is also known that $x_u / d \le x_{u,\,max} / d$ for under-reinforced sections. Substituting Eqs. 16.7 and 16.11 in Eq. 16.5, we have:

Curvature ductility $\mu = (\in_{uc} / \in_y)\,\{(1 - k) / (x_u/d)\}$ (16.14)

Substituting $\in_y$, k and (x_u/d) from Eqs.16.8, 9 and 12, respectively, and using $\in_{uc} = 0.0035$ and $E_y = 200000$ N/mm^2, we have:

$$\mu = (25200/0.87\, f_y)\,(f_{ck} / p\, f_y)\left\{1 + (mp/100) - \sqrt{(mp/100)^2 + 2(mp/100)}\right\} \quad (16.15)$$

From Fig. 16.40.5c, we also have

$$x_u / (d - x_u) = \in_{uc} / \in_{ust}$$

which gives

$$x_u / d = \in_{uc} / (\in_{uc} + \in_{ust}) \quad (16.16)$$

where $\in_{ust}$ = maximum strain in tension steel = $\mu_s \in_y$, where μ_s = Ductility in steel with respect to strain.

Thus, the curvature ductility $\mu = \phi_u/\phi_y$ (Eq. 16.5) can be determined in any of the following ways:

(i) Employing Eq. 16.14 using $\in_{uc} = 0.0035$ and determining $\in_y$, k and (x_u/d) from Eqs. 16.8, 16.9 and 16.12, respectively from the given f_{ck}, f_y and p.

(ii) Employing Eq. 16.15 from the given f_{ck} , f_y and p.

The value of k can be determined by finding the depth of the neutral axis taking moment of the compression concrete and the tensile steel about the neutral axis directly in place of employing Eq. 16.9. Numerical problems are solved in sec. 16.40.12 to illustrate the determination of ductility factor with respect to curvature.

The value of x_u/d can also be determined from Eq.16.16 if the value of μ_s, ductility in steel with respect to strain is given.

(B) Doubly-reinforced rectangular sections.

Doubly-reinforced rectangular sections have the area of compression steel = A_{sc} in addition to the area of tension steel = A_{st}. Accordingly, the value of k is obtained after determining the depth of the neutral axis by taking moment of the compression concrete, compression steel and tension steel about the neutral axis. The expression of x_u/d is derived below. Equating the total compressive force C due to concrete (= $0.36\, f_{ck}\, b\, x_u$) and compression steel (= $f_{sc}A_{sc}$) with the tensile force T (= $0.87\, f_y\, A_{st}$) for a doubly-reinforced rectangular section, we have:

$$0.36\, f_{ck}\, b\, x_u + f_{sc}\, A_{sc} = 0.87\, f_y\, A_{st} \tag{16.17}$$

which gives

$$\frac{x_u}{d} = \frac{0.87\, f_y A_{st}}{0.36\, f_{ck} bd} - \frac{f_{sc} A_{sc}}{0.36\, f_{ck} bd} \tag{16.18}$$

Using $A_{st} = p\, bd/100$ and $A_{sc} = p_c\, bd/100$ in Eq. 16.18, we have:

$$(x_u/d) = (f_y/36 f_{ck})\, \{0.87p - f_{sc}\, p_c / f_y\} \tag{16.19}$$

When $f_{sc} = 0.87\, f_y$ (16.20)

we have from Eq. 16.19:

$$(x_u/d) = (0.87\, f_y/36\, f_{ck})\, (p - p_c) \le (x_{u,\,max}/d) \tag{16.21}$$

Accordingly, the ductility factor with respect to curvature for the doubly-reinforced rectangular sections can be determined as explained through illustrative examples in sec. 16.40.12.

16.40.7 Factors Influencing Ductility

The following factors influence the ductility of reinforced concrete sections.

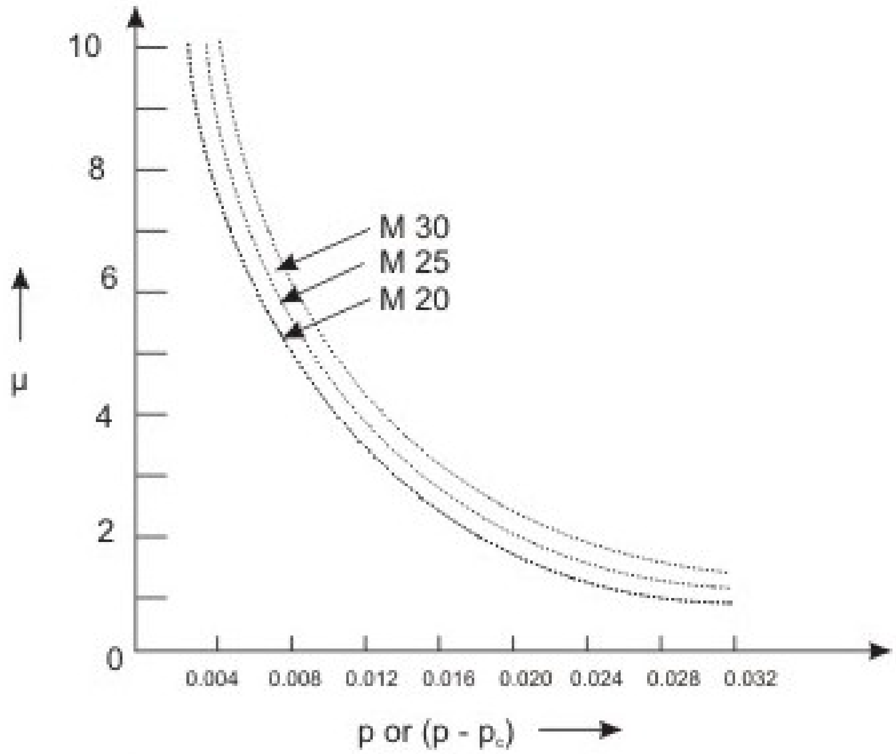


Fig. 16.40.6: Curvature ductility of beams (Fe 415)

(i) Figure 16.40.6 shows the plots of ductility μ with respect to curvature versus p or (p – p_c) for singly / doubly-reinforced rectangular sections for Fe 415 steel and three grades of concrete. It is evident from the plots that the ductility decreases with the increase of p or (p – p_c). Accordingly, cl. 6.2.2 of IS 13920:1993 stipulates the maximum percentage of steel on any face at any section as 2.5. Providing high percentage of tension steel will cause brittle failure by crushing of concrete. Thus, the sections should be designed as under-reinforced.

(ii) Equation 16.15 reveals that the ductility decreases with increasing grade of steel for a particular grade of concrete and specific percentage of steel. Hence, Fe 250 (mild steel) is the most preferred steel from the point of ductility of reinforced concrete sections. Clause 5.3 of IS 13920:1993 recommends steel reinforcement of grade Fe 415 or less. However, Fe 500 and Fe 550, produced by thermo-mechanical treatment process are also recommended by the code as they have elongation more than 14.5 per cent provided they conform to other requirement of IS 1786:1985.

(iii) Equation 16.15 and Fig. 16.40.6 show that the ductility increases with increasing grade of concrete for a particular grade of steel and specific percentage of steel. Accordingly, cl.5.2 of IS 13920:1993 prescribes the minimum grade of concrete as M 20 for all buildings which are more than three storeys in height.

(iv) Lower values of k and x_u will have higher values of ductility as evident from Eq. 16.14. Accordingly, T beams have more ductility than that of rectangular beams because of the reduced depth of neutral axis due to the presence of enlarged compression flange.

(v) The presence of lateral reinforcement prevents premature shear failure to enable the sections undergoing sufficient deformation. Moreover, lateral reinforcement in the compression zone arrests the buckling of compression reinforcement. Thus, lateral reinforcement, though cannot increase the ductility directly, helps to reach the attainable ductility as per the design.

16.40.8 Design for Ductility

The objectives of the ductile design of reinforced concrete members are to ensure both strength and ductility for the designed structures or members. Strength of members can be assured by proper design of the sections following limit state method as explained earlier. However, for ensuring ductility, specific recommendations are to be followed as given in IS 13920:1993 regarding the materials, dimensions, minimum and maximum percentages of reinforcement. Further, detailing of reinforcement plays an important role. Accordingly, some of the major steps to be followed in the design are given below which will ensure sufficient ductility in the design.

(i) General specification of materials

(a) The minimum grade of concrete shall be M 20 for all buildings, which are more than three storeys in height (cl. 5.2 of IS 13920:1993).

(b) Steel reinforcing bars of grade Fe 415 or less shall be used. However, steel bars of grades Fe 500 and Fe 550 may be used if they are produced by thermo-mechanical treatment process having elongation more than 14.5 per cent (cl. 5.3 of IS 13920:1993).

(ii) General guidelines in the design and detailing

(a) Simple and regular layout should be made avoiding any offsets of beams to columns or offsets of columns from floor to floor. Changes of stiffness of columns should be gradual from floor to floor.

(b) In a reinforced concrete frame, beams and columns should be designed such that the inelasticity is confined to beams only, while the columns should remain elastic. To satisfy this requirement, the sum of the moment capacities of the columns at a beam-column joint for the design axial loads should be greater than the sum of the moment capacities of the beams along each principal plane. Therefore,

$$\sum M_{\text{column}} \rangle \ 1.2 \ \sum M_{\text{beams}} \qquad (16.22)$$

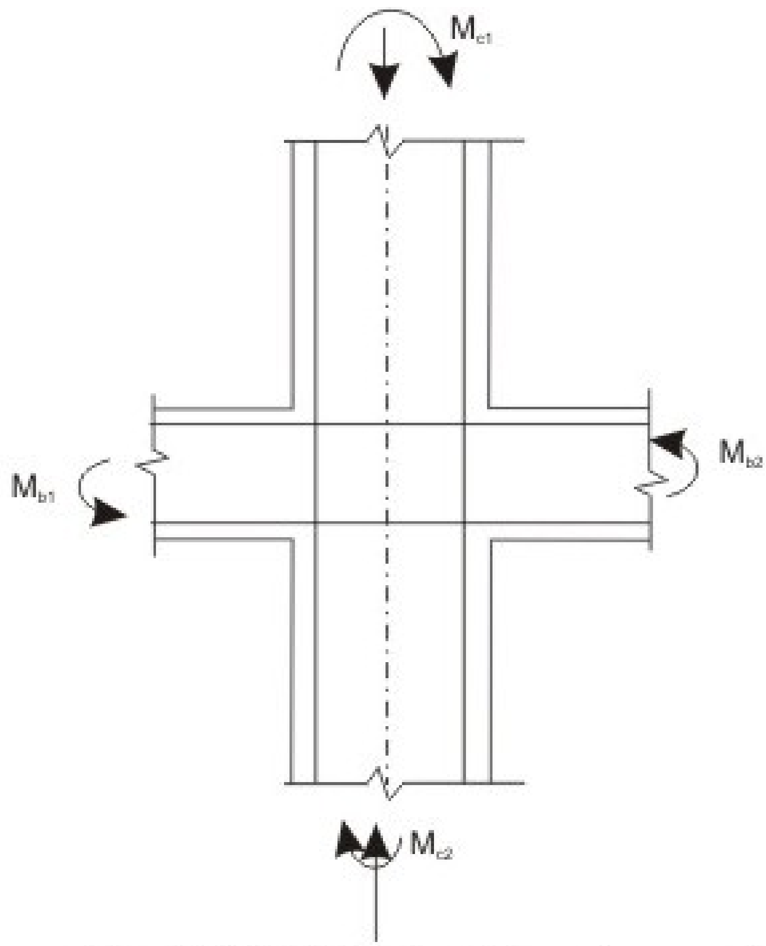


Fig. 16.40.7: Weak girder - strong column

The moment capacities of beams and columns are such that the column moments oppose the beam moments as shown in Fig. 16.40.7.

(c) The beams and columns should be designed taking into account the reversal of stresses due to the nature of earthquake forces.

(d) Beam-column connections should be made monolithic.

(e) The following are the requirements of flexural members:

- The factored axial stress on the member due to earthquake loading shall not exceed 0.1 f_{ck} (cl. 6.1.1 of IS 13920:1993).

- The width to depth ratio of the member shall preferably be more than 0.3 (cl. 6.1.2 of IS 13920:1993).

- The width of the member should not be less than 200 mm (cl. 6.1.3 of IS 13920:1993).

- The total depth *D* of the member shall preferably be not more than 1/4 of the clear span (cl. 6.1.4 of IS 13920:1993).

- The minimum percentage of tension steel on any face at any section is $24\sqrt{f_{ck}}/f_y$, where f_{ck} and f_y are in N/mm^2 (cl. 6.2.1b of IS 13920:1993).

- The maximum percentage of steel on any face at any section is 2.5 (cl. 6.2.2 of IS 13920:1993).

- The positive steel at a joint face must be at least equal to half the negative steel at that face (cl. 6.2.3 of IS 13920:1993).

- The redistribution of moments shall be used only for vertical load moments and not for lateral load moments (cl. 6.2.4 of IS 13920:1993).

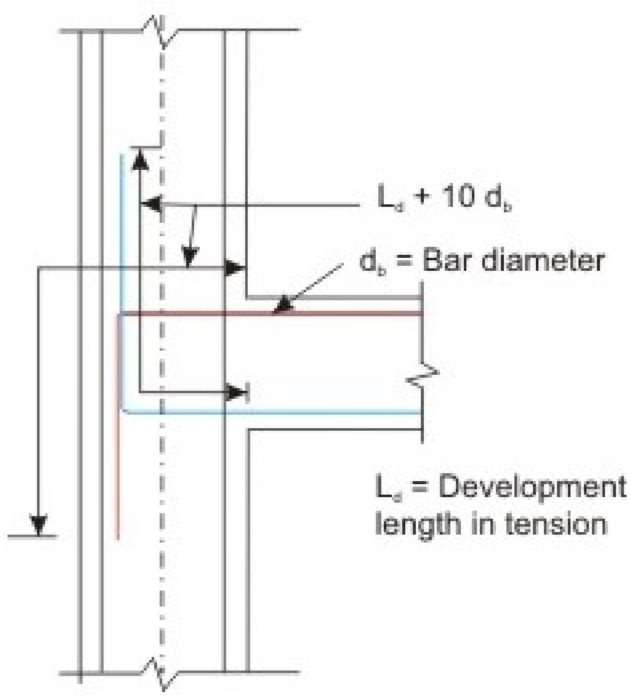
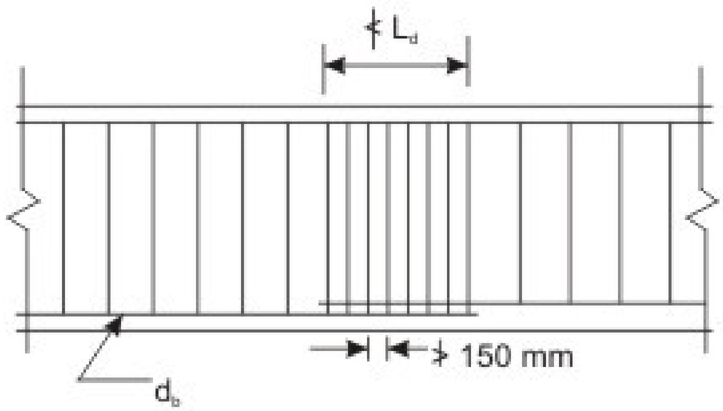


Fig. 16.40.8: Anchorage of beam bars in an external joint

- The anchorage length of top and bottom bars of the beam in an external joint shall be measured beyond the inner face of the column and should be equal to development length in tension plus 10 times the bar diameter minus the allowance for 90 degree bends (Fig. 16.40.8). In an internal joint, both face bars of the beam shall be taken continuously through the column (cl. 6.2.5 of IS 13920:1993).



L_d = Development length in tension
d_b = Bar diameter

Fig. 16.40.9: Lap splice in beam

- Longitudinal bars shall be spliced if hoops are provided over the entire splice length at a spacing not exceeding 150 mm, as shown in Fig. 16.40.9. The lap length shall be at least equal to the development length in tension. However, lap splices should not be provided (a) within a joint, (b) within a distance of 2*d* from the face of the joint, and (c) within a quarter length of the member where flexural yielding may generally occur under the effect of earthquake forces. Moreover, at one section not more than 50 per cent of the bars should be spliced (cl. 6.2.6 of IS 13920:1993).

16.40.9 Design for Shear in Flexural Members

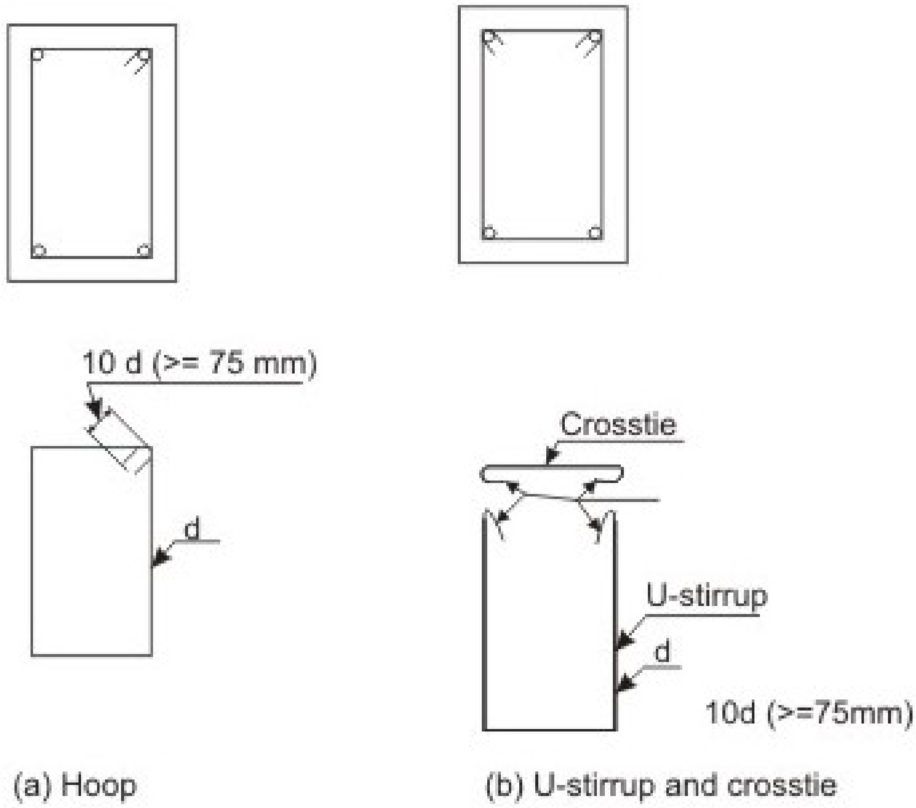


Fig. 16.40.10: Hoop, and U-stirrup and crosstie

Web reinforcement shall consist of either (a) vertical hoops or (b) U-stirrup with a 135 degree hook and a 10 diameter extension (but not 〈 75 mm) at each end and a crosstie, as shown in Figs. 16.40.10a and b, respectively. The crosstie is a bar having a 135 degree hook with a 10 diameter extension (but not 〈 75 mm) at each end (cl. 6.3.1 of IS 13920:1993). Normally, vertical hoops shall be used except in compelling circumstances when U-stirrups and crosstie shall be used.

The minimum diameter of the bar forming a hoop shall be 6 mm, except for beams with clear span exceeding 5 m, where the minimum diameter of the bar forming a hoop shall be 8 mm (cl. 6.3.2 of IS 13920:1993).

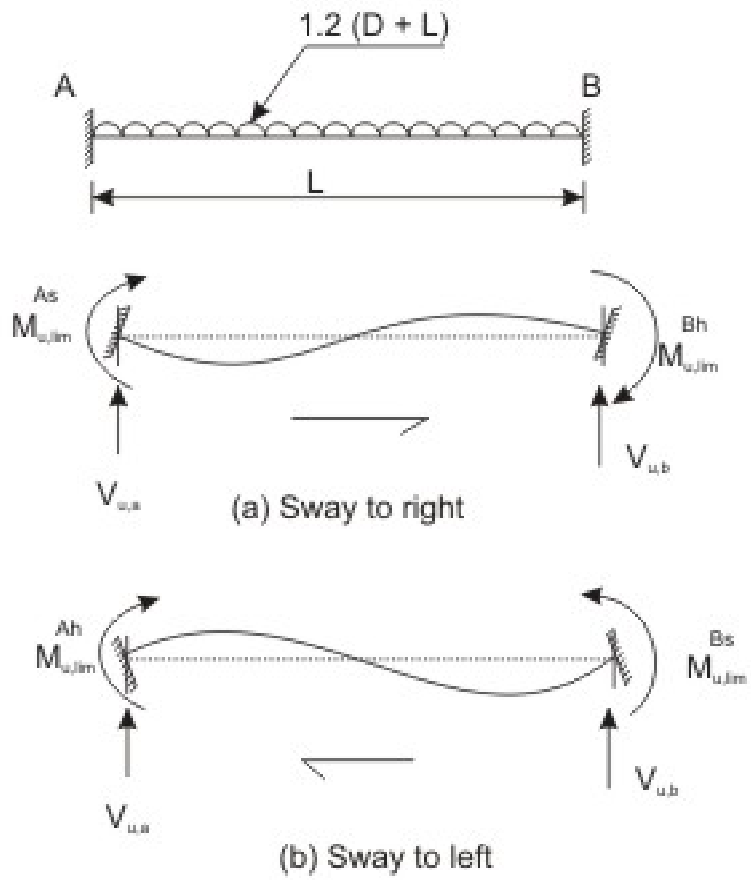


Fig. 16.40.11: Design shear force for beam

As per cl. 6.3.3 of IS 13920:1993, the shear force to be resisted by the vertical hoops shall be the maximum of:

(i) calculated factored shear force as per analysis, and

(ii) shear force due to formation of plastic hinges at both ends of the beam plus the factored gravity load on the span. The expressions are given below (Fig. 16.40.11 a and b):

(a) for sway to right:

$$V_{u,a} = V_a^{D+L} - 1.4\left[\frac{M_{u,lim}^{As} + M_{u,lim}^{Bh}}{L_{AB}}\right]$$

(16.23)

and

$$V_{u,b} = V_b^{D+L} + 1.4\left[\frac{M_{u,\lim}^{As} + M_{u,\lim}^{Bh}}{L_{AB}}\right]$$, and

(b) for sway to left

$$V_{u,a} = V_a^{D+L} + 1.4\left[\frac{M_{u,\lim}^{Ah} + M_{u,\lim}^{Bs}}{L_{AB}}\right]$$

(16.24)

and

$$V_{u,b} = V_b^{D+L} - 1.4\left[\frac{M_{u,\lim}^{Ah} + M_{u,\lim}^{Bs}}{L_{AB}}\right]$$

where

$M_{u,\lim}^{As}$, $M_{u,\lim}^{Ah}$ and $M_{u,\lim}^{Bs}$, $M_{u,\lim}^{Bh}$ = sagging and hogging moments of resistance of the beam section at ends A and B, respectively, L_{AB} = clear span of the beam, V_a^{D+L} and V_b^{D+L} = shear forces at ends A and B, respectively, due to vertical loads with a partial safety factor of 1.2 on loads.

Out of the two values of $V_{u,a}$, the design shear at the end A shall be the larger. Similarly, out of the two values of $V_{u,b}$, the design shear at the end B shall be the larger.

The expressions of Eqs. 16.23 and 16.24 are based on the assumption that the ratio of the actual ultimate tensile stress to the actual tensile yield strength of steel is not less than 1.25. Accordingly, 1.25 times the yield strength of steel divided by 0.87 is 1.43, which has been taken as 1.4 in Eqs. 16.23 and 16.24.

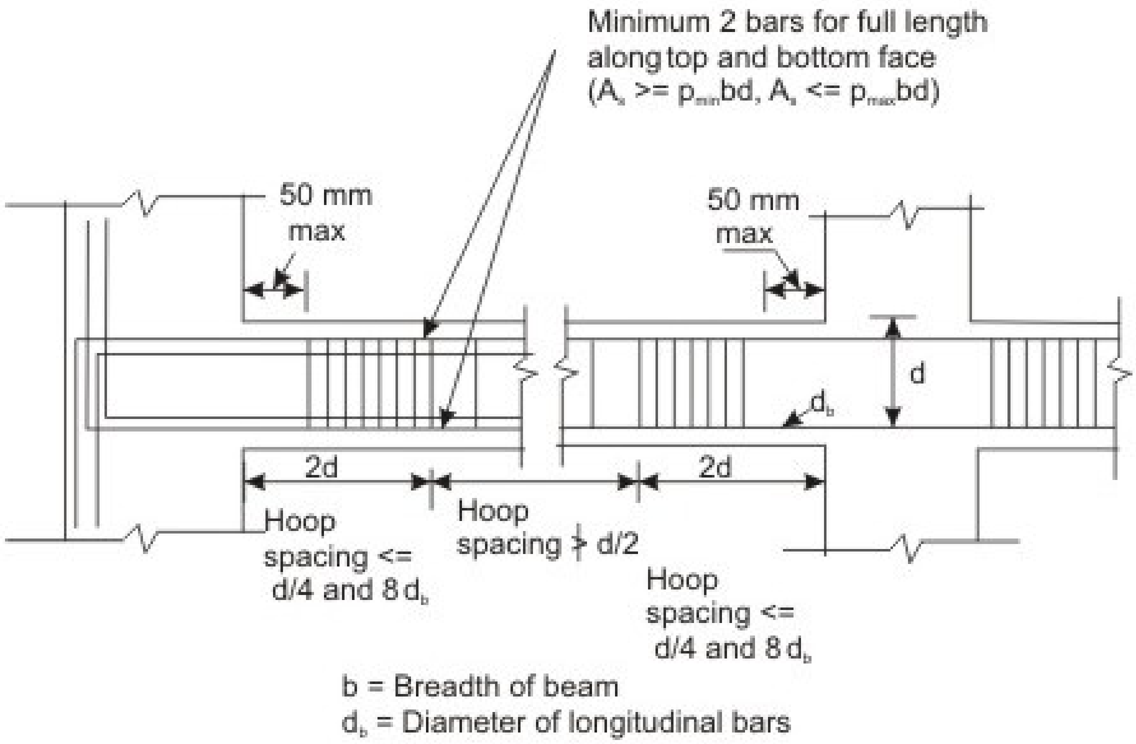


Fig. 16.40.12: Beam reinforcement

The contributions of bent up bars and inclined hoops, if any, shall not be considered to shear resistance of the section, as stipulated in cl. 6.3.4 of IS 13920:1993. Figure 16.40.12 shows the spacing of hoops and the following recommendations of cl. 6.3.5 of IS 13920:1993 are to be followed:

- Over a length of 2*d* at either end of the beam, the spacing of hoops shall not exceed (a) *d*/4, and (b) 8 times the diameter of the smallest longitudinal bar. However, the spacing shall not be less than 100 mm.

- The first hoop shall be at a distance not exceeding 50 mm from the joint face.

- Vertical hoops at the same spacing as mentioned above shall also be provided over a length of 2*d* on either side of a section where flexural yielding may occur under the effect of earthquake forces.

- At other places, the beam shall have vertical hoops at a spacing not exceeding *d*/2.

16.40.10 Column and Frame Members Subjected to Bending and Axial Load

The following requirements are applicable to frame members having factored axial stress more than 0.1 f_{ck} under the effect of earthquake forces.

The dimension of the member should be at least 200 mm. However, in frames having beams with centre-to-centre span exceeding 5 m or columns of unsupported length exceeding 4 m, the shortest dimension of the column shall be at least 300 mm (cl. 7.1.2 of IS 13920:1993).

(A) Longitudinal reinforcement

Longitudinal reinforcing bars shall be spliced only in the central half of the member length. Hoops shall be provided over the entire splice length at spacing not exceeding 150 mm centre to centre. A maximum of 50 per cent of the bars shall be spliced at one section (cl. 7.2.1 of IS 13920:1993).

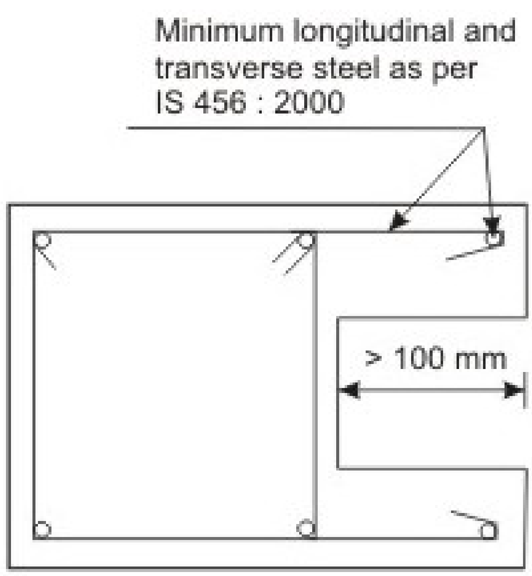


Fig. 16.40.13: Reinforcement for column with projection > 100 mm beyond core

The detailing of any area of column extending more than 100 mm beyond the confined core due to architectural requirement (cl. 7.2.2 of IS 13920:1993) shall be as follows:

(a) If the contribution of this area to strength is considered, then it should be the minimum longitudinal and transverse reinforcement as per IS 13920:1993.

(b) If this area is treated as non-structural, minimum longitudinal and transverse reinforcement shall be provided as per IS 456:2000, as shown in Fig. 16.40.13.

(B) Transverse reinforcement

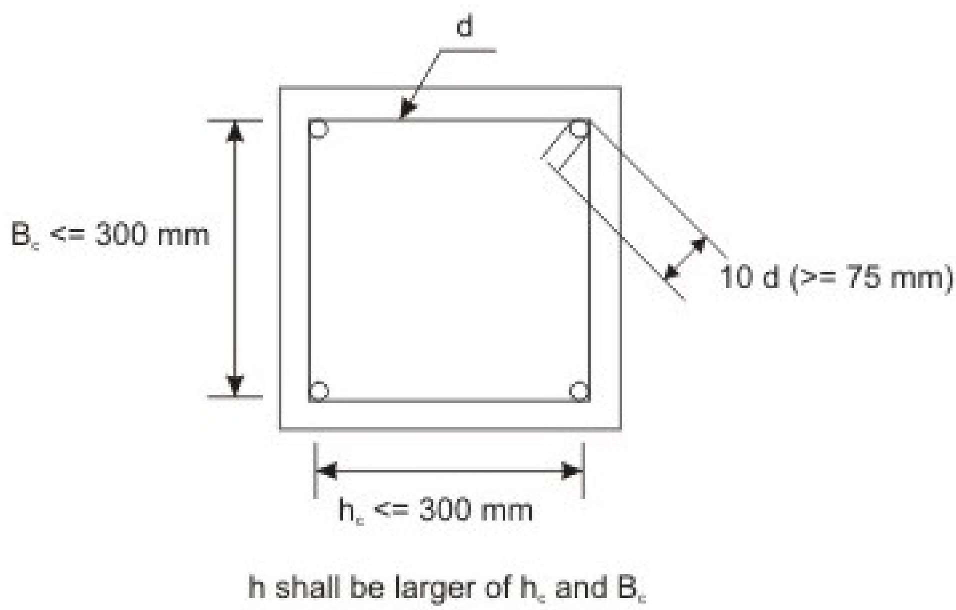


Fig. 16.40.14: Single hoop

Transverse reinforcement shall consist of spiral or circular hoops for circular columns and rectangular hoops for rectangular columns. Rectangular hoops shall be a closed stirrup having a 135 degree hook with a 10 diameter extension (but not 〈 75 mm) at each end, as shown in Fig. 16.40.14 (cl. 7.3.1 of IS 13920:1993).

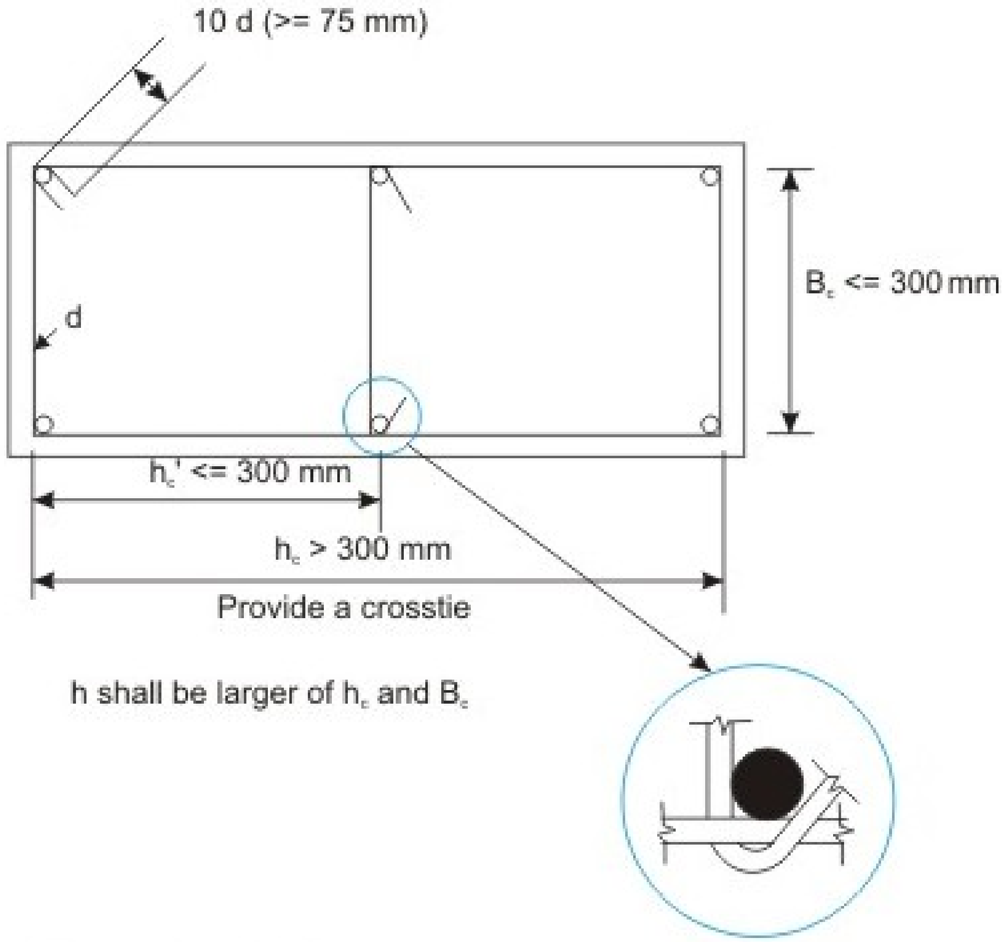


Fig. 16.40.15: Single hoop with a crosstie

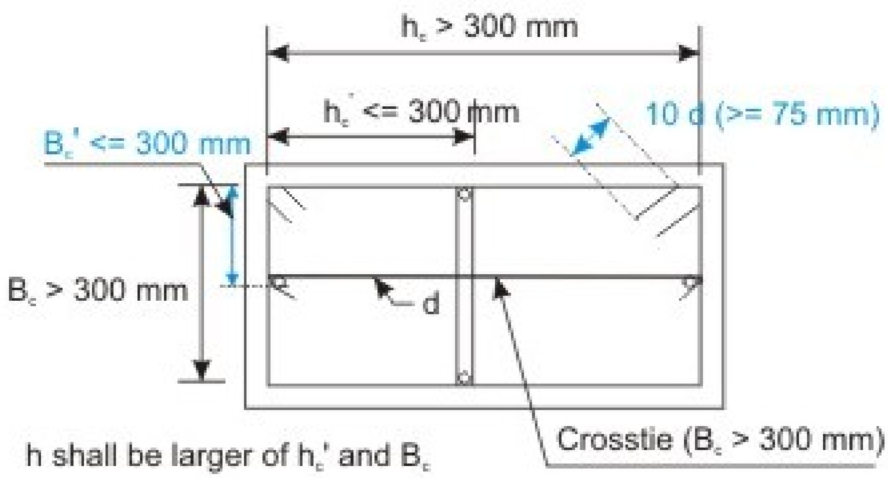
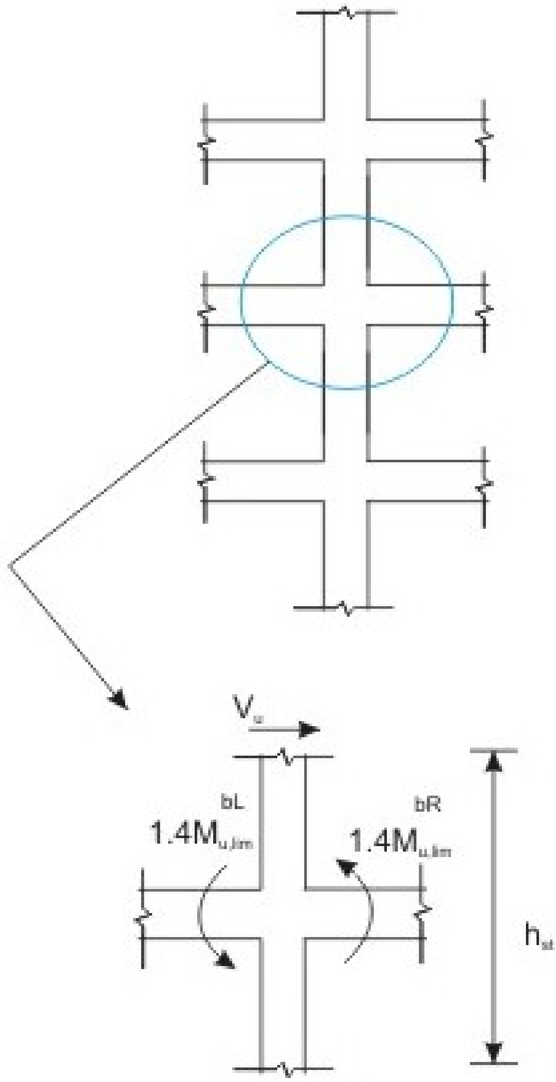


Fig. 16.40.16: Overlapping hoops with a crosstie

The parallel legs of rectangular hoop shall be spaced not exceeding 300 mm c/c. For spacing more than 300 mm, a crosstie shall be provided as shown in

Fig. 16.40.15. Alternatively, a pair of overlapping hoops may be provided within the column, as shown in Fig. 16.40.16. Hooks shall engage peripheral longitudinal bars (cl. 7.3.2 of IS 13920: 1993).

The maximum spacing of hoops shall be half the least lateral dimension of the column, except where spiral confining reinforcement is provided as per cl. 7.4 of IS 13920:1993 (cl. 7.3.3 of IS 13920:1993).



$$V_u = 1.4\left[\frac{M_{u,lim}^{bL} + M_{u,lim}^{bR}}{h_{st}}\right]$$

Fig. 16.40.17: Design shear force for column

The design shear force for columns, as recommended in cl. 7.3.4 of IS 13920:1993, shall be the maximum of:

(i) calculated factored shear force as per analysis, and
(ii) a factored shear force given by

$$V_u = 1.4\left[\frac{M_{u,lim}^{bL} + M_{u,lim}^{bR}}{h_{st}}\right] \tag{16.25}$$

where

$M_{u,lim}^{bL}$ and $M_{u,lim}^{bR}$ = moments of resistance, of opposite sign, of beams framing into the column from opposite faces, as shown in Fig. 16.40.17, and

h_{st} = storey height.

16.40.11 Special Confining Reinforcement

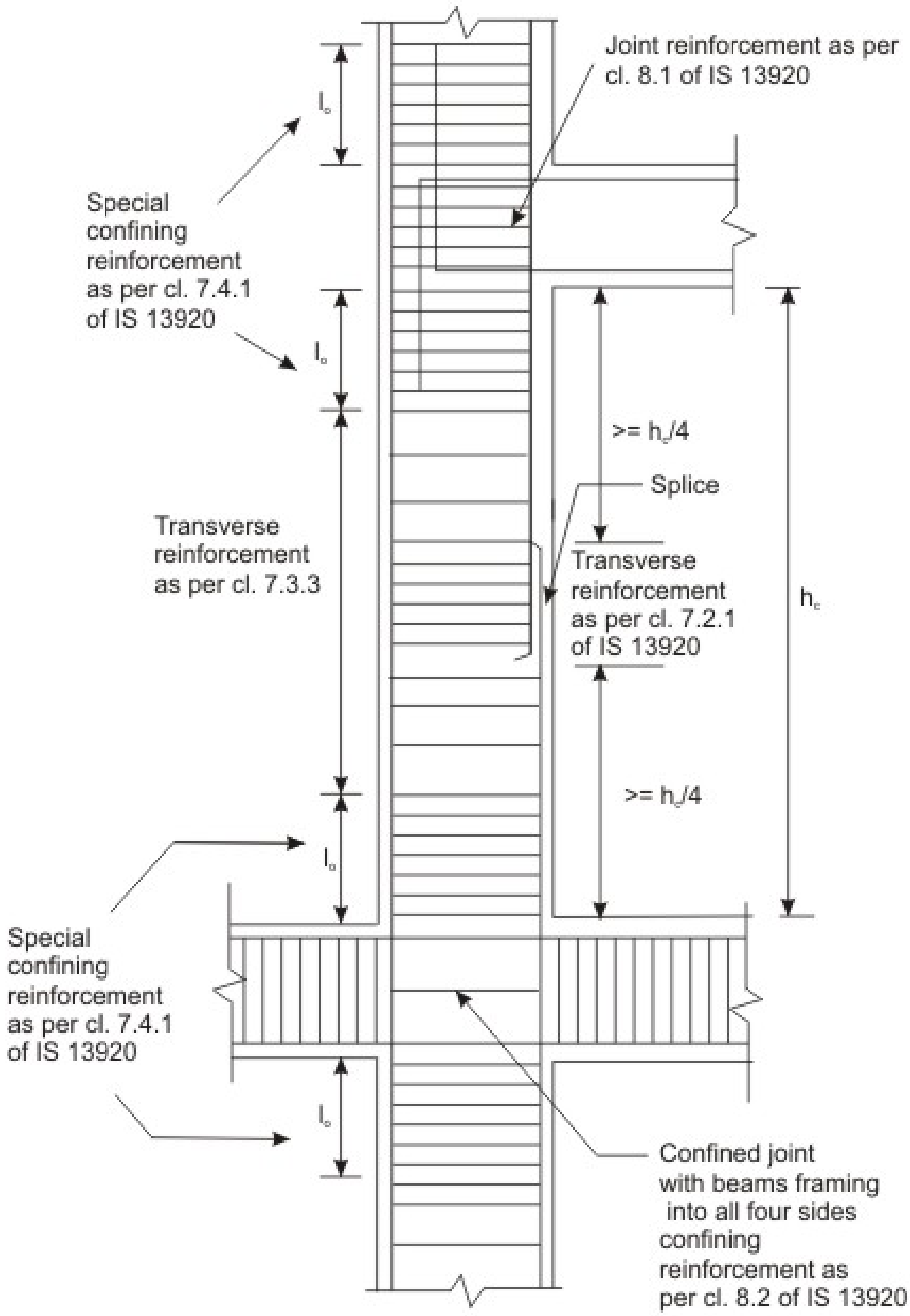


Fig. 16.40.18: Column and joint detailing

The following requirement shall be met with, unless a larger amount of transverse reinforcement is required from shear strength considerations.

Special confining reinforcement shall be provided over a length l_o from each joint face towards mid-span, and on either side of any section, where flexural yielding may occur under the effect of earthquake forces, as shown in Fig. 16.40.18. The length l_o shall not be less than (a) larger lateral dimension of the member at the section where yielding occurs, (b) 1/6 of clear span of the member, and (c) 450 mm (cl. 7.4.1 of IS 13920:1993).

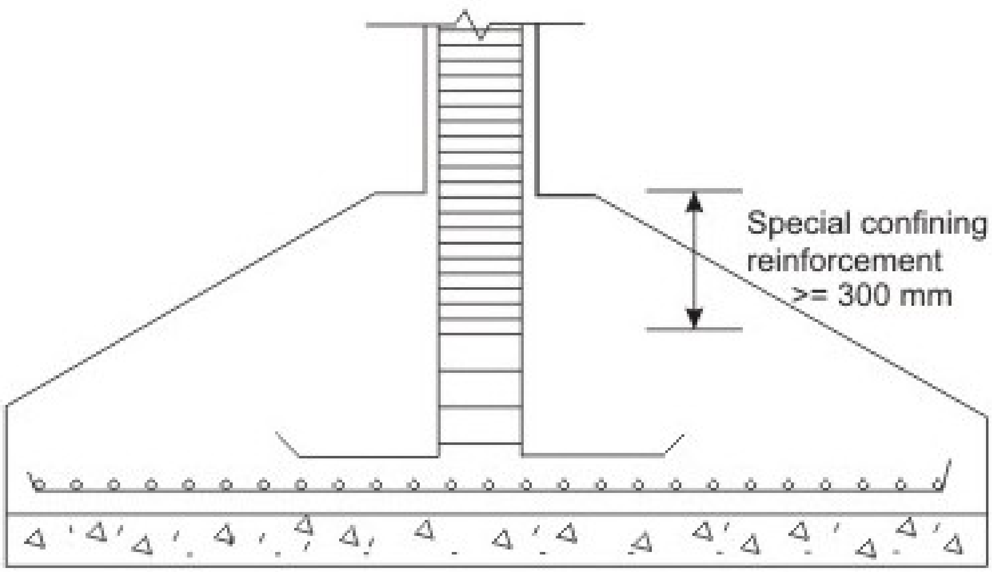


Fig. 16.40.19: Special confining reinforcement in footimg

Special confining reinforcement shall extend at least 300 mm into the footing or mat when a column terminates into a footing or mat, as shown in Fig. 16.40.19 (cl. 7.4.2 of IS 13920:1993).

Special confining reinforcement shall be provided over the full height of the column for the following situations (cls. 7.4.3 to 5 of IS 13920:1993):

(i) when the calculated point of contra-flexure, under the effect of gravity and earthquake loads, is not within the middle half of the member clear height,

(ii) columns supporting reactions from discontinued stiff members, such as walls, and

(iii) columns which have significant variation in stiffness along their heights.

The maximum spacing of hoops used as special confining reinforcement shall be 1/4 of minimum member dimension. However, the spacing of hoops should not be less than 75 mm nor more than 100 mm (cl. 7.4.6 of IS 13920: 1993).

The area of cross-section A_{sh} of the bar forming circular hoops or spiral, to be used as special confining reinforcement, shall not be less than (cl.7.4.7 of IS 13920:1993).

$$A_{sh} = 0.09 \; S \, D_k \, \frac{f_{ck}}{f_y} \left[\frac{A_g}{A_k} - 1 \right] \tag{16.26}$$

where

A_{sh} = area of the bar cross-section,
S = pitch of spiral or spacing of hoops,
D_k = diameter of core measured to the outside of the spiral or hoop,
f_{ck} = characteristic compressive strength of concrete cube,
f_y = yield stress of steel (of circular hoop or spiral),
A_g = gross area of column cross-section, and
A_k = area of concrete core = $(\pi/4)\, D^2_k$

The area of cross-section A_{sh} of the bar forming rectangular hoop, to be used as special confining reinforcement, shall not be less than (cl.7.4.8 of IS 13920:1993):

$$A_{sh} = 0.18 \; S \, h \, \frac{f_{ck}}{f_y} \left[\frac{A_g}{A_k} - 1.0 \right] \tag{16.27}$$

where

h = longer dimension of the rectangular confining hoop measured to its outer face. It shall not exceed 300 mm (see Figs. 16.40.14, 15 and 16), and

A_k = area of confined concrete core in the rectangular hoop measured to its outside dimensions.

16.40.12 Numerical Problems

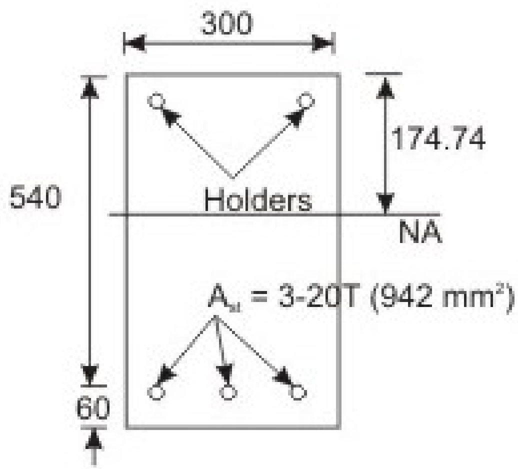


Fig. 16.40.20: Problem 1

Problem 1: Determine the ductility with respect to curvature of the beam shown in Fig. 16.40.20 using M 20 and Fe 250.

Solution 1: This is a singly-reinforced rectangular beam of b = 300 mm, d = 540 mm, D = 600 mm, A_{st} = 942 mm^2 (3-20T), f_{ck} = 20 N/mm^2 and f_y = 250 N/mm^2.

Step 1: Checking for minimum and maximum percentages of reinforcement (sec. 16.40.8 (ii) e)

Minimum percentage of A_{st} = 100 (0.24) $\sqrt{f_{ck}}$ / f_y = 0.429

Maximum percentage of A_{st} = 2.5

For this problem p = 94200/(300) (540) = 0.581. Hence, A_{st} is within minimum and maximum percentages i.e., 0.429 < 0.581 < 2.5).

Step 2: Determination of $\in_y$, k and x_u/d (Eqs. 16.8, 16.9 and 16.12)

From Eq. 16.8, $\in_y = f_y / E_s$ = 250/200000 (1)

From Eq. 16.9, $k = -(mp/100) + \{(mp/100)^2 + 2\,(mp/100)\}^{1/2}$ where $m = 280/3\sigma_{cbc}$ = 280/3(7) = 13.33 and p = 0.581. Therefore, k = 0.3236.

Taking moment of compression concrete and tension steel about the neutral axis, we have: 300 (x^2/2) = 942 (13.33) (d - x), or $x^2 + 83.7124x - 45204.696 = 0$, which gives $x = 174.742$ mm. Therefore, $k = x/d = 0.3236$.

From Eq. 16.12, $x_u/d = 0.87\, f_y\, p/(36 - f_{ck}) \le (x_{u,\,max}/d)$, which gives: $x_u/d = 0.87(250)\,(0.581)/(36)\,(20) = 0.1755 < 0.53$, (as $x_{u,\,max}/d = 0.53$).

Step 3: Determination of ductility with respect to curvature (Eq. 16.14)

From Eq. 16.14, $\mu = (\in_{uc} / \in_y)\ \{(1 - k) / (x_u/d)\}$. Using $\in_{uc} = 0.0035$ and substituting the values of $\in_y$, k and (x_u/d) from step 2,

$\mu = \{0.0035\ (200000)/250\}\ \{(1 - 0.3236) / 0.1755\} = 10.79$

Hence, the ductility of this beam $\mu = 10.79$.

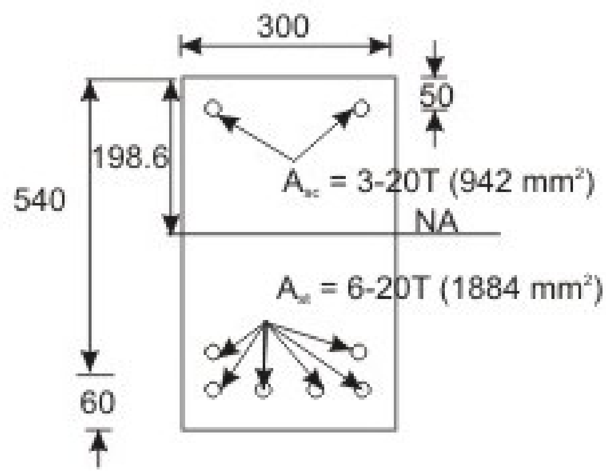


Fig. 16.40.21: Problem 2

Problem 2: Compare the ductility with respect to curvature of the cross-section of the beam of Fig. 16.40.21 using (a) M 20 and Fe 250, and (b) M 20 and Fe 415.

Solution 2: This is a doubly-reinforced rectangular beam of b = 300 mm, d = 540 mm, D = 600 mm, d' = 50 mm, A_{st} = 1884 mm^2 (6-20T), A_{sc} = 942 mm^2 (3-20T), f_{ck} = 20 N/mm^2 and f_y = 250 N/mm^2 for (a); and 415 N/mm^2 for (b). The amounts of steel are within minimum and maximum percentages. Here, p = 1.162 and p_c = 0.581. The modular ratio m for M 20 for both (a) and (b) parts of the problem = 13.33 as in Problem 1.

Step 1: **Determination of *k* for both parts a and b**

Taking moment of compression concrete, compression steel and tension steel about the neutral axis, we have:

$300\ (x^2/2) + 942\ (1.5\ m - 1)\ (x - 50) = 1884\ (13.33)\ (d - x)$
or $x^2 + 286.7134x - 96373.822 = 0$, which gives: $x = 198.5862272$ mm. Hence, $k = x/d = 198.5862272/540 = 0.3677$

Step 2: **Determination of x_u/d (Eq. 16.21)**

Equation 16.21 is: $x_u/d = (0.87\ f_y/36\ f_{ck})\ (p - p_c) \le (x_{u,\ max}/d)$

For (a), when f_y = 250 N/mm^2:
$x_u/d = \{0.87\ (250)/36(20)\}\ (0.581) = 0.1755 < 0.53$, (as $x_{u,max}/d$ = 0.53)

For (b), when f_y = 415 N/mm^2:
$x_u/d = \{0.87\ (415)/36(20)\}\ (0.581) = 0.2913 < 0.48$, (as $x_{u,max}/d$ = 0.48)

Step 3: **Determination of ductility with respect to curvature (Eq. 16.14)**

Equation 16.14 is: $\mu = (\in_{uc}\ /\ (\in_y)\ \{(1 - k)\ /\ (x_u\ /\ d)\}$. Using $\in_{uc}$ = 0.0035, $\in_y = f_y/E_s$, and substituting the values of k and x_u/d from steps 1 and 2, we have,

For (a): $\mu = \{(0.0035)\ (200000)/250\}\ \{(1 - 0.3677)\ /\ 0.1755\} = 10.088$

For (b): $\mu = \{(0.0035)\ (200000)/415\}\ \{(1 - 0.3677)\ /\ 0.2913\} = 3.66$

Hence, the ductility of this doubly-reinforced beam is 10.088 when Fe 250 is used and is 3.66 when Fe 415 is used. The loss of ductility when opting for Fe 415 in place of Fe 250 is noted.

Problem 3: Design an inner beam of span 6 m of one reinforced concrete frame for ductility using M 25 and Fe 415. The beam has negative bending moment of 300 kNm and shear force of 250 kN at the face of beam-column joint due to gravity and earthquake loads.

Solution 3:

Step 1: **Selection of dimensions and determining areas of steel**

Let us assume the dimensions b = 300 mm and D = 600 mm. With effective cover of 55 mm, d = 545 mm and d'/D = 0.1 and using partial safety factor for load as 1.2, we have: M_u/bd^2 = $1.2(300)10^6/300\ (545)\ (545) = 4.04$ N/mm^2

The design for ductility has been given in sec. 16.40.8 (ii) following the general guidelines in the design and detailing. Following the general guidelines, Design Tables 20.2 and 20.3 have been presented in the book, "Reinforced Concrete Limit State Design" (6th Edition) by A.K. Jain. These tables give the percentages of tension and compression steel for M 20 and M 25 grades up to $p/p_c \le 2.0$ for four different values of d'/D as 0.05, 0.10, 0.15 and 0.20, and M_u/bd^2. Accordingly, we refer to Table 20.3 for this problem, when M_u/bd^2 = 4.04 N/mm^2 and d'/D = 0.1 to get p = 1.2554 and p_c = 0.6276.

The minimum percentage of steel = 100 (0.24) $\sqrt{25}$ / 415 = 0.289 and maximum percentage of steel = 2.5. Thus, the determined percentages are within the respective limits. So, A_{st} = 1.2554 (300) (545) / 100 = 2052.58 mm^2. Provide 2-28T + 2-25 T (1231 + 981 = 2212 mm^2) giving p provided = 1.353 per cent. The compression steel A_{sc} = 0.6276 (300) (545)/100 = 1026.13 mm^2. Provide 2-28T (1232 mm^2), giving p_c provided = 0.753 per cent.

Step 2: Design for shear

With partial safety factor for loads as 1.2, the factored shear force V_u = 1.2 (250) = 300 kN.

$\tau_v = V_u/bd$ = 300/(0.3) (545) = 1.83 N/mm^2,

τ_c = (from Table 19 of IS 456 with p = 1.353%) = 0.716 N/mm^2,

and τ_{max} (from Table 20 of IS 456) = 3.1 N/mm^2

Since $\tau_c < \tau_v < \tau_{max}$, we have to provide vertical hoops (stirrups). Providing 10 mm–2 legged hoops (A_{sv} = 157 mm^2), we have the spacing of hoops as

$S_v = 0.87\, f_y\, A_{sv}\, d/(V_u - \tau_c\, bd) = 0.87\, f_y\, A_{sv} / (\tau_v - \tau_c)b$ = 0.87(415) (157) / (1.83 – 0.716) 300 = 169.61 mm.

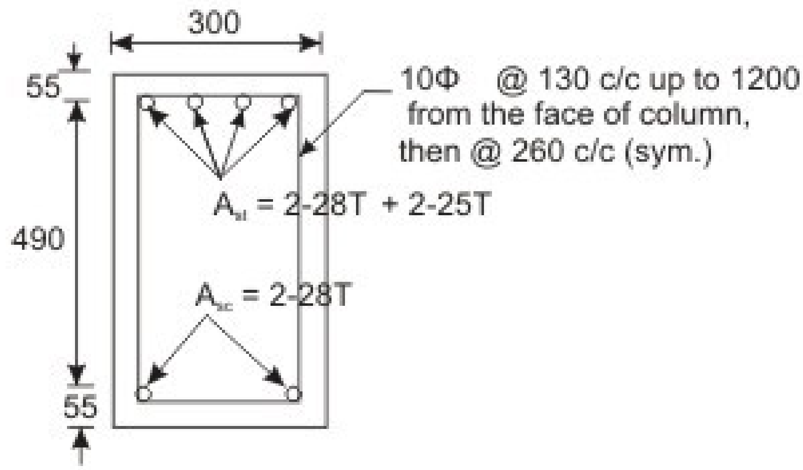


Fig. 16.40.22: Problem 3

The arrangement, minimum and maximum spacing of hoops are given in sec. 16.40.9. The maximum spacing is (a) *d*/4 = 136.25 mm, (b) 8 (25) = 200 mm and the spacing shall not be less than 100 mm, for a length of 2 *d* at either end of the beam. So, provide 10 mm – 2 legged hoops @130 mm c/c up to 1200 mm from the face of the column and then @ 260 mm c/c up to the centre-line, to be placed symmetrically. It may be noted that the maximum spacing at other places is *d*/2 = 272 mm. The first hoop shall be provided at a distance of 30 mm (< 50 mm as per the code) from the face of the column. There will be ten hoops in 1200 mm. Figure 14.40.22 shows the cross-section of the beam.

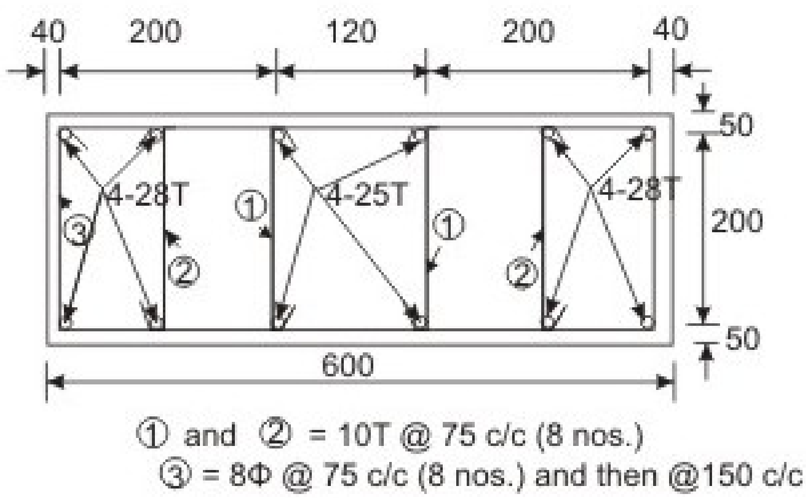


Fig. 16.40.23: Problem 4

Problem 4: Check (i) if the inner beam-column joint of a reinforced concrete frame satisfies weak girder–strong column proportion and (ii) shears in beam and column, using M 25, Fe 415 and other details as given below:

Clear span of the left beam = 5 m, clear span of the right beam = 4 m, slab thickness = 120 mm, finish on slab = 50 mm, live loads on floor = 2.0 kN/m^2, axial load on column = 800 kN, beam dimensions = 300 mm $\times$ 600 mm, steel bars at top on either side = 4-25T + 2-20T (1.6 per cent), steel bars at bottom on either side = 2-25T + 1-20T (0.8 per cent), column dimensions = 300 mm $\times$ 600 mm, column reinforcement bars = 8-28T + 4 -25T (3.827 per cent), and height of storey = 3.6 m (Fig. 16.40.23).

Solution 4:

(i) Beam-column Joint

As given in sec. 16.40.8 (ii), the requirement of the design is to satisfy Eq. 16.22, i.e.,

$$\sum M_{column} \rangle 1.2 \sum M_{beam}$$

along each principal plane. We consider the joint in bending about the weaker axis and use tables of SP-16 for the computation.

Assuming d'/d = 0.1 and with p = 1.6%, Table 51 of SP-16 gives M_u/bd^2 = 4.77 for doubly-reinforced section. This gives the hogging moment capacity as M_u = 4.77 (300) (545) (545) = 425.04 kNm. The sagging moment capacity of singly-reinforced section is obtained from Table 3 of SP-16, where M_u / bd^2 = 2.503 for p = 0.8%. Thus, M_u = 2.503 (300) (545) (545) = 223.04 kNm. Therefore,

$$\sum M_{beam} = 425.04 + 223.04 = 648.08 \text{ kNm}$$

For columns P_u = 1.2 (800) = 960 kN gives $P_u/f_{ck}\, bD$ = 1.2 (800) / (25) (3) (60) = 0.213 and p / f_{ck} = 3.827/25 = 0.153. Chart 32 of SP-16 gives $M_u/f_{ck}\, bD^2$ = 0.256. The column moment M_u = 0.256 (25) (300) (600) (600) = 691.2 kNm. Therefore,

$$\sum M_{column} = 2\,(691.2) = 1382.4 \text{ kNm.}$$

Here, $\sum M_{column} \rangle (1.2) \sum M_{beam}$ (= 1.2 (648.08) = 777.7 kNm)
Hence, the requirement of Eq. 16.22 is satisfied.

(ii) Shear in beam and column

Step 1: Shear capacity of left beam having spacing of 4m c/c

Live loads = 4 m (2 kN/m^2) = 8 kN/m
Dead load of slab = 4 (0.12 + 0.05) (25) = 17.0 kN/m
Dead load of web of beam = 0.3 (0.48) (25) = 3.6 kN/m

Total dead load = 20.6 kN/m.

Factored shear force due to gravity loads = 1.2 (20.6 + 8) (5)/ 2 = 85.8 kN.

Factored shear force due to plastic hinge = 1.4 (648.08) / 5 = 181.46 kN

Thus, V_u = 85.8 + 181.46 = 267.26 kN

τ_v = 267.26 / 300 (0.545) = 1.63 N/mm^2
τ_c (from Table 19 of IS 456 for p = 1.6 %) = 0.756 N/mm^2.
τ_{cmax} (from Table 20 of IS 456) = 3.1 N/mm^2.

Since $\tau_c < \tau_v < \tau_{cmax}$, we provide 8 mm-2 legged hoops (A_{sv} = 100 mm^2) of spacing s_v where s_v is as follows:

$s_v = 0.87\, f_y\, A_{sv} / (\tau_v - \tau_c)b$ = 0.87 (415) (100) / (1.63 – 0.756) (300) = 137.7 mm c/c.

For a distance of 2 d (= 1090 mm) from the face of the column, the maximum spacing is: (a) 0.25 (545) = 136.25 mm and (b) 8 (20) = 160 mm. The spacing should not be less than 100 mm. Beyond 1090 mm, the spacing = d/2 = 272.5 mm.

Let us change the diameter of hoop as 10 mm beyond 1090 mm, for which the spacing is 0.87 (415) (157)/(1.63 – 0.756) (300) = 216.36 mm c/c.

So, let us provide 8 mm-2 legged hoops @ 130 mm c/c up to 1.2 m from the face of the column at either end. The first hoop shall be at a distance of 30 mm from the face of the joint. We need ten hoops. Beyond 1.2 m, provide 10 mm-2 legged hoops @ 200 mm c/c, symmetrically.

Step 2: Checking of column for storey height = 3600 mm

Factored shear in column V_u = 1.4 (648.08)/3.6 = 252.03 kN. This gives τ_v = 252.03/3 (60) = 1.4 N/mm^2.

τ_c (from Table 19 of IS 456: 2000 with 1.913% A_{st} at each face) = 0.806 N/mm^2. This shall be multiplied by a factor = 1+3 (1.2) (800)/(30) (6) (25) = 1.64. However, the multiplying factor is limited to 1.5 (see cl. 40.2.2 of IS 456: 2000). So, τ_c = 1.5 (0.806) = 1.209 N/mm^2.

τ_{cmax} (from Table 20 of IS 456: 2000) = 3.1 N/mm^2. Since $\tau_c \langle \tau_v \langle \tau_{cmax}$, we provide 8 mm – 2 legged hoop of mild steel (Fe 250), for which the spacing is

$s_v = 0.87 f_y A_{sv}/(\tau_v - \tau_c) b$ = 0.87(250)(100)/(1.4 – 1.209) (300) = 379.58 mm.

The maximum spacing is 0.5 (300) = 150 mm (cl. 7.3.3 of IS 13920:1993). Hence, provide 8 mm-2 legged hoops of Fe 250 @150 mm c/c except in the confining steel zone.

Step 3: Confining steel

Here, axial stress = 1.2 (800) / 30(6) = 5.33 N/mm^2 $\rangle$ 0.1 f_{ck} (=2.5 N/mm^2). So, we provide confining steel of rectangular closed loops of spacing lesser of (a) 0.25 (300) and (b) 100 mm (cl.7.4.6 of IS 13920: 1993). So, the spacing is 75 mm c/c. From Fig. 16.40.23, we have h = 200 mm.

From Eq. 16.27, we have: $A_{sh} = 0.18 \; S\, h \{(A_g / A_k) - 1\} (f_{ck} / f_y)$
= 0.18 (75) (200) [(300) (600) / (184) (504)}-1](25/415) = 153.05 mm^2

Provide 2-10 mm rectangular hoops (157 mm^2). These confining hoops shall be provided for a distance greater of: (i) 600 mm, (ii) 3600/6 = 600 mm and (c) 450 mm. So, we provide 8 numbers of 2 legged 10 mm diameter hoops @ 75 mm c/c up to 630 mm from the joint. The first one is at a distance of 30 mm from the joint (see sec.16.40.11).

16.40.13 Practice Questions and Problems with Answers

Q.1: Define and explain ductility with respect to (a) displacement, (b) curvature and (c) rotation of a reinforced concrete structure.

A.1: Secs. 16.40.2, 3 and 4 are the respective answers of part (a), (b) and (c).

Q.2: State the advantages of ductility in reinforced concrete structures.

A.2: Sec. 16.40.5 is the complete answer.

Q.3: Derive the expressions of ductility of (a) singly-reinforced and (b) doubly-reinforced concrete beams.

A.3: Part (A) and Part (B) of sec. 16.40.6 are the answers of (a) and (b), respectively.

Q.4: State the factors influencing ductility of reinforced concrete sections.

A.4: Sec. 16.40.7 is the complete answer.

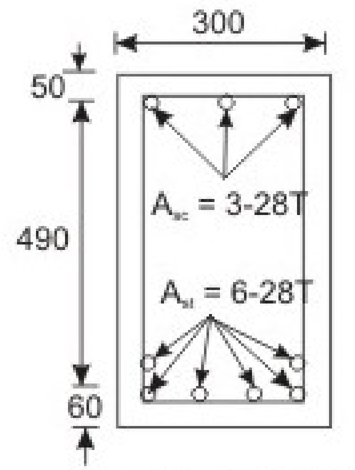


Fig. 16.40.24: Q.5

Q.5: Compare the ductility with respect to curvature of the cross-section of the beam of Fig. 16.40.23 using (a) M 25 and Fe 250, and (b) M 25 and Fe 415.

A.5: This is a doubly-reinforced rectangular beam of b = 300 mm, d = 540 mm, D = 600 mm, d' = 50 mm, A_{st} = 3694 mm^2 (6-28T), A_{sc} = 1847 mm^2 (3-28T), f_{ck} = 25 N/mm^2 and f_y = 250 N/mm^2 for (a) and 415 N/mm^2 for (b). The percentages of tension steel p = 2.28% and p_c = 1.14%. The reinforcing steel is within the range of minimum and maximum percentages. The modular ratio $m = 280/3\sigma_{cbc} = 280/3(8.5) = 10.98$.

Step 1: Determination of *k* for both parts a and b

Taking moment of compression concrete, compression steel and tension steel about the neutral axis, we have:

$$300\,(x^2/2) + 1847(1.5\,m - 1)\,(x-50) = 3694\,(m)\,(d-x)$$

or $x^2 + 460.8880667x - 155540.7953 = 0$

or x = 226.333 mm giving $k = x/d$ = 0.419

Step 2: Determination of x_u/d (Eq. 16.21)

Equation 16.21 is: $x_u/d = (0.87\,f_y/36\,f_{ck})\,(p - p_c) \le (x_{u,\,max}\,/d)$

For (a), when f_y = 250 N/mm^2: x_u/d = {0.87 (250) / 36(25)} {1.14} = 0.2755 $\langle$0.53

(as $x_{u,max}$ / d = 0.53)

For (b), when f_y = 415 N/mm^2: x_u/d = {0.87 (415) / 36(25)} {1.14} = 0.457 $\langle$0.48

(as $x_{u,max}$ / d = 0.48)

Step 3: Determination of ductility with respect to curvature (Eq. 16.14)

Equation 16.14 is: $\mu = (\epsilon_{uc} / \epsilon_y) \{(1- k)/(x_u / d)\}$
Using ϵ_{uc} = 0.0035, $\epsilon_y = f_y / E_s$ and substituting the values of k and x_u /d from steps 1 and 2, we have

For (a): μ = {(0.0035) (200000)/250} {(1 – 0.419) / (0.2755)} = 5.905
For (b): μ = {(0.0035) (200000)/415} {(1 – 0.419) / (0.457)} = 2.144

Hence, the ductility of this doubly-reinforced beam is 5.905 when Fe 250 is used and is 2.144 when Fe 415 is used.

Q.6: Design the column of a mulitstoreyed building for ductility with M 25 and Fe 415 subjected to an axial force of 2000 kN and bending moment of 416.67 kNm.

A.6:

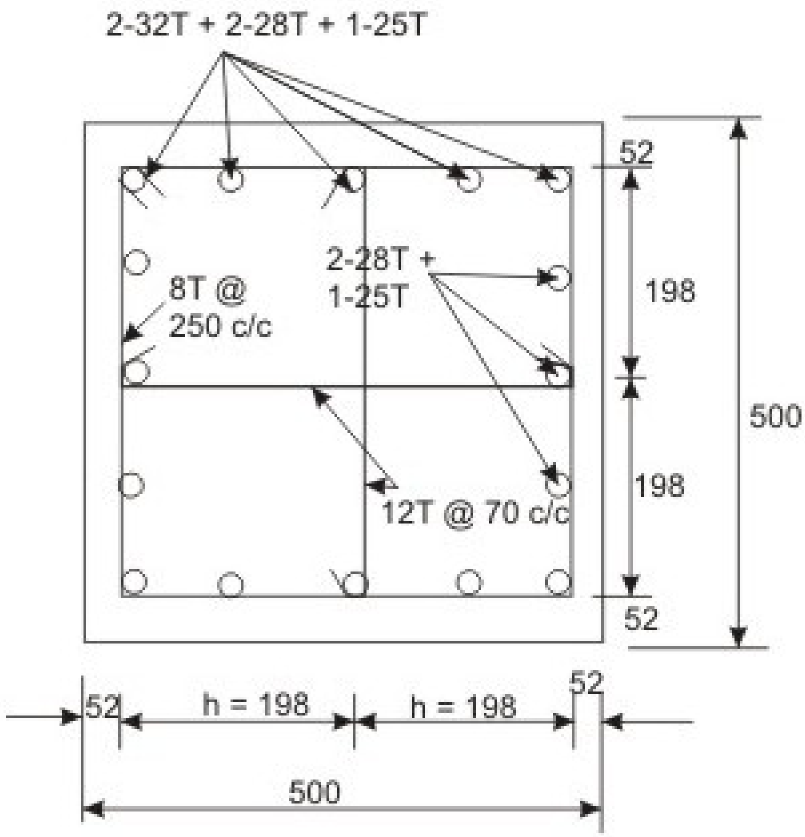


Fig. 10.40.25: Q. 6

Step 1: Dimensions and area of steel of column

Let us take the column dimensions as 500 mm × 500 mm. Considering partial safety factor for loads = 1.2, we have:

M_u = (1.2) (416.67) = 500 kNm
P_u = (1.2) (2000) = 2400 kN

$P_u/f_{ck}\, bD$ = 2400 / (25) (5) (50) = 0.384
$M_u/f_{ck}bD^2$ = 500 / (25) (125) = 0.16
d'/D = 75/500 = 0.15

Chart 45 of SP-16 gives p/f_{ck} = 0.16. With p = 0.16 (25)= 4%, we have A_{st} = 4(500)(500)/100 = 10000 mm^2. Provide 4 - 32T + 8 – 28T + 4 – 25T (= 3217 + 4926 + 1963 = 10106 mm^2) as shown in Fig. 16.40.25.

Step 2: Lateral ties

Diameter of lateral tie is the greater of 32/4 = 8 mm or 6 mm. Let us take 8 mm diameter bars.

Maximum pitch is 0.5 (500) = 250 mm (cl. 7.3.3 of IS 13920:1993). So, provide 8 mm ties @ 250 mm c/c.

Step 3: Confining reinforcement

Here, P_u/A = 2400 (10^3)/(500) (500) = 9.6 N/mm^2 $\rangle$ 0.1 f_{ck} (=2.5 N/mm^2). Hence, confining reinforcement shall be provided.

From Eq. 16.27, we have

$$A_{sh} = 0.18\ S\ h\ \frac{f_{ck}}{f_y}\left[\frac{A_g}{A_k} - 1\right]$$

A_{sh} = 113 mm^2 for 12 mm diameter hoop.
h = 500 – 2 (40 + 12) = 396 mm $\rangle$300 mm.
So, h is revised as 396/2 = 198 mm $\langle$ 300 mm.
A_g = 500 (500) = 250000 mm^2
A_k = {500 – 2 (40 + 12 + 8)} (380) = 144400 mm^2 and {(A_g/A_k) – 1} = 0.7313.

Hence, S = 113 (415)/{0.18(198) (0.7313) (25)} = 71.9 mm

Provide confining hoops of 12 mm diameter bar @ 70 mm c/c, as shown in Fig. 16.40.25.

The distance of the confining reinforcement is the largest of: (i) clear span of column / 6 = 3600/6 = 600 mm, (ii) larger lateral dimension = 500 mm and (iii) 450 mm. So, provide confining hoops for a distance of 700 mm from the face of the joint (eleven numbers of hoop).

16.40.14 References

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2. Limit State Design of Reinforced Concrete, 2nd Edition, by P.C. Varghese, Prentice-Hall of India Pvt. Ltd., New Delhi, 2002.
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16.40.15 Test 40 with Solutions

Maximum Marks = 50 Maximum Time = 30 minutes

Answer all questions.

TQ.1: State the advantages of ductility in reinforced concrete structures.

(10 Marks)

A.TQ.1: Sec. 16.40.5 is the complete answer.

TQ.2: Derive the expressions of ductility of (a) singly-reinforced and (b) doubly-reinforced concrete beams.

(15 Marks)

A.TQ.2: Part (A) and Part (B) of sec. 16.40.6 are the answers of (a) and (b), respectively.

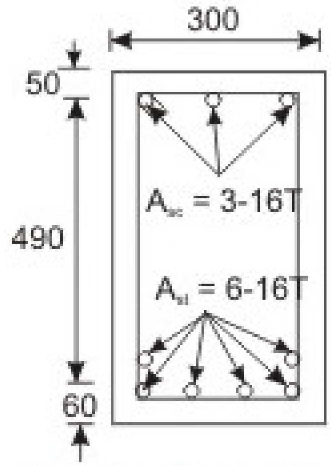

Fig. 16.40.26: TQ. 3

TQ.3: Compare the ductility with respect to curvature of the cross section of the beam of Fig. 16.40.25 using (a) M 30 and Fe 250 and (b) M 30 and Fe 415.

A.TQ.3: This is a doubly-reinforced rectangular beam of b = 300 mm, d = 540 mm, D = 600 mm, d' = 50 mm, A_{st} = 1206 mm^2 (6 – 16T), A_{sc} = 603 mm^2 (3 – 16T), f_{ck} = 30 N/mm^2 and f_y = 250 N/mm^2 for (a) and 415 N/mm^2 for (b). The modular ratio m = 280/3(10) = 9.33, minimum percentage of steel = $24\sqrt{30}\,/\,250$ = 0.526 for (a) and = $24\sqrt{30}\,/\,415$ = 0.317 for (b). Here, p = 120600/300(540) = 0.7444 per cent. Thus, percentage of A_{st} is within the range of minimum and maximum limits (2.5%).

Step 1: Determination of *k* for both parts (a) and (b)

Taking moment of compression steel and tension steel about the neutral axis, we have:

$$300\,(x^2/2) + 603(1.5\,m\text{-}1)\,(x - 50) = 1206\,(m)\,(d - x)$$

or $$x^2 + 127.2531x - 43119.123 = 0$$

which gives x = 153.554 mm, which gives: $k = x/d$ = 0.284.

Step 2: Determination of x_u/d (Eq. 16.21)

Equation 16.21 is: $x_u/d = (0.87\, f_y /36\, f_{ck})\,(p - p_c) \le (x_{u,\,max}/d)$

For (a), when f_y = 250 N/mm^2: x_u/d = {0.87 (250) / 36(30)} (0.3722) = 0.0749 $\langle$
0.53 (as $x_{u,max}$ / d = 0.53)

For (b), when f_y = 415 N/mm^2: x_u/d = {0.87 (415) / 36(30)} (0.3722) = 0.124 $\langle$ 0.48 (as $x_{u,max}$ / d = 0.48)

Step 3: Determination of ductility with respect to curvature (Eq. 16.14)

Equation 16.14 is: $\mu = (\epsilon_{uc} / \epsilon_y) \{(1 - k) / (x_u/d)\}$

Using ϵ_{uc} = 0.0035, $\epsilon_y = f_y / E_s$ and substituting the values of k and x_u / d from Steps 1 and 2, we have:

For (a): μ = {(0.0035) (200000)/250} {(1 – 0.284) / (0.0749)} = 26.766
For (b): μ = {(0.0035) (200000)/415} {(1 – 0.284) / (0.124)} = 9.74

Hence, the ductility of this doubly-reinforced beam (Fig. 16.40.26) is 26.766 when Fe 250 is used and is 9.74 when Fe 415 is used.

16.40.16 Summary of this Lesson

This lesson defines and explains the ductility factor with respect to displacement, curvature and rotation. Stating and explaining the advantages of ductility in the design of reinforced concrete members, expressions of ductility factor are derived for singly and doubly-reinforced concrete rectangular beams. Factors influencing the ductility are explained. General specifications of materials are given and the general guidelines in the design and detailing of structures are stated as stipulated in IS 13920:1993. The situations requiring special confining reinforcement are explained. Several illustrative examples are solved determining the ductility of singly and doubly-reinforced rectangular beams. Furthermore, numerical problems are also solved to explain the design of beams, columns and beam-column joints as per IS 13920:1993.